The Hicks Bargain:
A Rational Reconstruction

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Abstract

The paper provides a rational reconstruction of Hicks largely intuitive account of his theory of the wage bargain of 1932. The implication of its Hicksian reconstruction for the wage rate bargain struck by a trade union and business confederation is compared with the corresponding Nash and Kalai-Smorodinsky solutions. The wage rate comparative-statics of the Hicksian reconstruction are explored for what they imply for wage rigidity.
The Problem of the Bargain

The problem of the explaining the bargain struck between a single seller and single buyer was first scientifically broached by the inaugural generation of English neoclassicals; Jenkin, Edgeworth, and Marshall (Creedy1988 44-45); but without much effect.¹

The second wind in bargaining theory came in the inter-war period with the almost simultaneous publications of two similar theories of the bargain by Hicks (1932) and Zeuthen (1930). These works advanced the first models that proposed a determinate solution. This was, however, something of a false-start; both books were surprisingly neglected in the Imperfect Competition Revolution of the 1930s, and nothing much issued further from these publications.

Nash (1950) launched a fresh research program – Axiomatic Bargaining Theory. Whereas the approach of both Hicks and Zeuthen was to characterise the logic of the bargaining process, the axiomatic bargaining approach worked through positing the properties that any outcome of a bargain should have. The program has prospered in the last 60 years, with an active exploration of alternative axioms that a bargaining outcome should conform to.

This paper turns a light back on Hicks’ bargaining theory.² It seeks to rationalise Hicks largely intuitive account of his theory. After expounding a ‘general’ interpretation, it applies the interpretation to the particular case of the wage bargain, explores wage rate comparative-statics implied by the interpretation, and compares what Hicksian model implies for the wage rate with the Nash and Kalai-Smorodinsky solutions.

¹ The lack of progress is underlined by how little Pigou in 1905 could offer by way of explaining the wage rate in a bilaterally monopolised labour market (Pigou 1905). Pigou did little more than establish that there existed a lower bound to the wage rate, below which both profit and labour’s surplus would fall; and an upper bound to the wage rate, above which both profit and labour’s surplus would fall.

²Some discussion of Hicks theory occur in Harsanyi (1956), Rothschild (1994), and Shackle (1957). Zeuthen’s theory has been expounded and critically appraised by a former pupil (Brems 1976), and Harsanyi (1956)
Historical Genesis

It is not fortuitous that Hicks theory appeared in the inter-war period.\(^3\)

In the present interpretation, Hicks’ analysis begins with the observation that there are two ways to settle some clash of wills. The two parties can agree; or they can fight it out. Hicks’ analysis, then, is concerned with the rational choice between those two alternatives; the rational choice we must often make between accord and affray.

I strongly suspect that Hicks’ sensitivity to this choice between accord and affray was fostered by the shadow cast by First World War. Throughout that War both sides felt they had a choice to make between fighting for victory, or, instead, negotiating peace. Throughout the fighting each side showed itself prepared to talk peace, and each proposed peace terms through intermediaries, such as President Wilson. The war ended, recall, on both eastern and western fronts, with peace treaties (Litovsk, Versailles).\(^4\)

The significance of the First World War can be underlined by contrasting it with the Second. In the later War there was no possibility of concluding the war through a negotiated peace. The credibility of such negotiations had been destroyed by the

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\(^3\) Neither is it fortuitous that Zeuthen’s did appear in this period.

\(^4\) ‘December 12 1916 The German government appealed to the U.S. to inform the Entente governments that the Central powers were prepared to negotiate peace…. President Wilson transmitted his own proposals to the warring powers. He suggested that the belligerents state their terms for peace and for arrangements to guarantee the world against renewal of conflict. The German, Austrian, and Turkish governments replied (Dec. 26) in an appreciative way, but reiterated their opinion that the best method would be to call a meeting for exchange of views. No definite terms were mentioned. The Allied powers in their reply (Jan. 10, 1917) named specific terms. These included the restoration of Belgium, Serbia, and Montenegro; the evacuation of French, Russian, and Romanian territory, with just reparation; the reorganization of Europe on the basis of nationalities; the restoration of territory previously taken from the Allies; the liberation of Italians, Slavs, Romanians, and Czechoslovaks from foreign rule; the freeing of subject nationalities under Turkish rule; and the expulsion of the Turks from Europe. The far-reaching nature of the Allied terms, at a moment when the military situation was by no means in their favor, estranged even Wilson, who still stuck by the idea of “peace without victory” (speech to the Senate, Jan. 22)’.
spread of the war through the spectacular repudiation of various non-aggression pacts and Hague treaties. Consequently, the conflict soon became a quasi-extirpatory ‘total’ war, a war of ‘unconditional surrender’; a war that could only end with the conquest of one side by the other. For each belligerent the only option was Victory. The belligerents of the first World War, by contrast were confronted with a genuine choice, between Victory or Peace.\(^5\) And that is the choice Hicks was concerned to understand.

**The elements of a reconstruction**

Hicks’ account of his theory (1932, pp140-146) is brief, verbal and informal. It includes a single figure, but no mathematics. To the present author it ‘seems reasonable’, without the reasoning being very clear. It is a representation of an intuition, rather than a proof of some theorem. This section of the paper attempts to obtain a rationalisation of Hicks by advancing a certain interpretation of Hicks, that deploys some of the moves that a modern theorist might make; including references to ‘symmetry’, ‘belief equilibria’ and ‘credible threats’. In order to get more effectively at rationalisation, we begin by casting the bargaining problem in quite general terms, and then proceeding to wage bargaining.

It would not be controversial to suggest that at the root of the logic of choice between Peace and Victory, lies the fact that there are pros and cons to both alternatives. Victory consumes resources in war, but results in the plunder of the other party at its conclusion. Peace economises the resources spent in war, but forgoes the spoils of ultimate victory. Thus Victory may be preferred to Peace, or Peace may be preferred to Victory.

To get a grip on these preferences, we make some particular assumptions. We suppose there is some object of dispute between two parties, \(Y\) and \(X\). It might be a

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\(^5\) In international relations between the wars it was very apparent that the world faced a choice between Wilsonian methods (no aggression, self-determination, sacredness of treaties) and various barbarian philosophies of power.
border that divides a certain valuable territory between two countries. This dispute can be concluded by Agreement or by Struggle. That is, to pursue our illustration in terms of international relations, the dispute can be concluded the by either treaty or war.

We assume any Struggle ends with the ‘complete victory’ for one side, in the sense that the victor is free to impose whatever terms it likes, and to its maximum advantage. In the example we have been using, the victor wins the entire territory, the loser loses the lot.⁶

We assume that the winner of struggle is the party that devotes the more resources to the struggle; if you like, the side that spends more on its armed forces. We also suppose the amount of resources is a choice variable. It is not that a side ‘endowed’ with some given amount of ‘struggle resources’. Rather, any amount of struggle resources can be purchased at a fixed price; and the price is the same for both parties. Thus the ‘cost of victory’ for any side is how much it believes it must spend to outspend its adversary in a war.

If Victory has a cost, Peace also has its price. Any peace terms will involve awarding at least some territory to the other side; perhaps only an epsilon amount, but perhaps much more. The ‘price of peace’ for any side is what that side forgoes by the peace terms relative to what it would have obtained by victory. The general point is that the peace terms might be severe, or they might be generous, but will always have some ‘price’; something forgone relative to victory.

It is evident that, for any given side, there are combinations of the terms peace and the cost of victory that will leave that side indifferent between Peace on those terms, and Victory. If we suppose the peace terms are measured by the number a so that the greater a, the greater the price of peace to Y, then ‘the greater’ the peace terms the

⁶ It simplifies to suppose that the territory’s value is not reduced by Struggle, that is, it is not damaged by war. This simplification can be relaxed without changing qualitative conclusions. Notice also that Victory is treated as purely a means to an end: it is not ‘its own reward’ in any way. There is no value that is attached to Victory as such.
greater the cost of victory to Y that leaves Y indifferent between Peace and Victory. Figure 1 shows this. Figure 1 plots the peace terms to Y on the vertical axis, and the cost of victory to Y on the horizontal axis. If $a$ is small, and peace is cheap to Y – if the terms offered are such that Y forgoes very little by agreeing- then the amount Y would be willing to pay for outright victory is very small. But if $a$ is large and peace is expensive – if the terms are so harsh to Y that Y is forgoing very much - then the amount Y would be willing to pay for outright victory is very large.

**Figure 1: Y’s Concession Curve**

![Diagram of Y’s Concession Curve]

In Hicksian terminology Figure 1 plots the *concession curve*. This curve indicates the maximum amount Y would be willing to pay to have victory rather than those peace terms.

The same exercise can be done for X, but since the interests of Y and X are directly conflicting – and the greater the ‘price’ of any peace to Y the smaller the price of that peace to X - the concession curve for X slope downwards.

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7 * $a$ might, for example, be the amount of the disputed territory awarded to X.
Hicks principal contention seems to be that from the data represented in Figures 1 and 2 we can derive an equilibrium peace terms, and an equilibrium belief in the cost of victory, shared by both countries.

**Equilibrium**

We begin by observing that since X and Y have exactly the same opportunities to win – they both face the same price of ‘struggle resources’ – it seems sensible that in equilibrium both must have the same belief as to the amount of resources that will secure victory. Thus we do not have to distinguish between the cost of victory to X and the cost of victory to Y; there is just one, commonly shared estimate of the Cost of Victory, and so the concession curves of X and Y can be plotted on the same figure, and in doing so we can obtain the plotting of preference of Y and X over peace and war implied by various peace terms and costs of victory (Figure 3).
Can we obtain combinations of peace terms and the cost of victory that would constitute an equilibrium?

Quadrants I, III and IV seem to provide only non-equilibria combinations.

If both sides are to agree on peace terms then both must consider the terms are better than war, but in all points Quadrants I, III and IV at least one party thinks they could do better by paying the cost necessary to win a war.

But war is not an equilibrium either. Consider Quadrant IV, that may suggest war. For any point in Quadrant IV what Figure 3 states is that Y would be better off by refusing the peace terms implied by that point, and spending the amount implied by that point in the pursuit of victory, on the assumption that that indicated amount would secure victory. But can that assumption be true? If Y believed it to be true, then
X would believe it to be true. But if they both believed that spending that amount would achieve victory then spending that amount would not achieve victory.

Quadrant I may also suggest war. At any point in Quadrant I Y judges they would be better off by refusing the peace and spending the amount indicated by the Cost of Victory corresponding to that point. X, by contrast, does not consider that Victory, acquired Cost of Victory corresponding to the point, superior to Peace. But while to X’s mind Peace is preferred to Victory, Victory is still preferred to defeat. So if it is ‘war’, X will spend to win. But if both spend the indicated amount then neither outspends the other, and neither achieves victory. There is no ‘equilibrium in beliefs’ here.

What about Quadrant II? Here we have both sides preferring peace to Victory for a great range of peace term. Here there are apparently and abundance of peace term possibilities. But one can argue that the lack of ‘credible threats’ disqualify all the points on II as disequilibria. Take any point in Quadrant II. What Figure 3 states is that Y would be better off by accepting the peace terms implied by that point rather than spending the amount implied by that point in the pursuit of victory. Y might embrace the peace terms corresponding to that point. But what is Y’s alternative? Their only alternative is a war in which victory is so costly, that winning it would make them worse off. Therefore there is no credible threat to the effect, ‘If you don’t accept this division of the territory proposed by the peace terms, then we are content go to war and take it all off you’. There is no credible threat to the effect, ‘We are offering these terms; and if you don’t take them, you’ll be sorry’.

We conclude that the only equilibrium seems to be at the intersection of the concession curves. At the peace terms implied by that intersection Victory is not preferred to Peace, at the corresponding combination of peace terms and victory cost;

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8 That is, the value of victory exceeds the cost of victory. The value of victory to X can be measured as the horizontal axis intercept of X’s concession curve, as that intercept indicates the amount X would be willing to spend to obtain victory rather than receive absolutely nothing; and that must be the value of victory. In being offered nothing X is asking themselves; ‘How much is the whole territory worth? That is eat I am willing to pay to conquer it.’

9 War in Quadrant III seems to be disqualified as an equilibrium by a comparable argument.
so peace can survive. Peace is not preferred to Victory: and that lends some credibility to the peace terms; it is not undermined by the belief that the other side would not be content to go to war rather than accept some other peace terms. Each side can credibly say to the other, ‘If you don’t accept this division, then we are content to go to war and take it all off you’.

An algebraic representation

It might be useful to give an example. To represent the concession curves in algebraic terms,

let $a$ be the index of the peace terms, and vary from 0 to 1; where $a = 0$ is the most favourable to Y, and $a = 1$ is the most favourable to X;

let $P_Y(a)$ be payoff of peace to Y, conditional on terms, $a$, where $P'_Y(a) < 0$. Similarly for X;

let $V_Y =$ payoff of victory to Y, $V_Y = P_Y(0)$. Similarly for X;

Then we can infer the cost of victory to Y, $C_Y$, that would leave Y between securing victory and accepting the peace terms; that is the $C_Y$ that equals the ‘price of peace’,

$$C_Y = V_Y - P_Y(a)$$

$$C_X = V_X - P_X(a)$$

We have argued that equilibrium occurs at the $a$ where

$$C_X = C_Y$$

or, equivalently, where the price of peace is equal,

$$V_Y - P_Y(a) = V_X - P_X(a)$$
Evidently, at the equilibrium what Y forgoes by accepting the terms rather than achieving victory equals what X forgoes by accepting the terms rather than achieving victory.

A numerical illustration might be helpful. Suppose Y is more efficient utiliser of territory than X. Suppose Y obtains twice the value as X from any square kilometre of ground. Then,

\[ P_Y(a) = 2[1 - a] \]
\[ P_X(a) = a \]
\[ V_Y = 2 \]
\[ V_X = 1 \]
\[ C_Y = 2 - 2[1 - a] = 2a \]
\[ C_X = 1 - a \]

Thus the solution is

\[ a = 1/3 \]

Y gets 2/3 of the territory, and X gets 1/3 of the territory.

Note also

\[ P_Y = 4/3 \]
\[ P_X = 1/3 \]

Y produces 4 times the value of X.

The Hicks Bargain in a wage determination context
Hicks expounded his theory in terms of a bargain over the wage between employees and employers. Let’s cast the interpretation of the Hicks bargain we have advanced in terms of a bargain specifically over wages.

We suppose that output $Y$ is produced by two factors, $L$ and $K$, according to a constant returns production function.

$$ Y = Kq \left( \frac{L}{K} \right) \quad q' > 0 ; q'' < 0 $$

The supply of capital is fixed, and the supply of labour is fixed at $\Sigma$. We suppose labour is represented by a Union, and capital owners by a Confederation. And the dispute is over the wage rate, $w$. Once the dispute is settled decisions over the volume of employment remain the province of the individual capital owning member of the business confederation, and conform to a standard labour demand curve, $l(w)K$. If the wage rate is settled above the competitive equilibrium, unemployment will result and we assume that is randomly allocated amongst members of the labour supply.

The Union and Confederation can either agree over $w$, or resort to a struggle. Victory for the business confederation will entail a wage rate of zero; that is the most advantageous wage rate to the business confederation, and it gets the whole of output. Victory for the Union is a wage that maximises the wage bill (ie the expected wage income of each worker). We denote this wage $w_{MON}$.

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10 $l(w) = q^{-1}(w)$

11 In the context of the standard monopoly equilibrium this would be the, and this is the wage rate such that the elasticity of demand for labour is 1, where the elasticity of labour demand is a positive function of the wage rate, and is less than 1 at the competitive wage rate. But we are not restricting the present analysis to that context. The elasticity of labour demand could be parametrically less than 1, in which case $w_{MON}$ is the wage that reduces the profit rate to zero. The elasticity of labour demand could be parametrically greater than 1, in which case $w_{MON}$ is the competitive wage.
What constitutes the ‘struggle’ here? Perhaps the most meaningful analogy to war, that is not very distant from ‘war’, is a political struggle; a struggle for control of the coercive, legislative power of the state. The dimensions of this struggle can be numerous. It could include campaigns to win public opinion, bribes for politicians, election campaigns.

If the Confederation wins that struggle and seizes control of the state, it passes a law reducing the wage rate to zero. If the Union wins that struggle and seizes control of the state, it passes a law imposes a wage rate equal to the wage bill maximising wage rate. So,

\[ V_U = w_{\text{MON}} l(w_{\text{MON}})K \]

\[ V_C = Kq \left( \frac{\sum}{K} \right) \]

If the wage is dispute is settled by agreement then the payoff for workers is \( wl(w)K \), and the payoff for capital owners is \( q(l(w)) - wl(w)K \). Thus,

\[ P_U(w) = wl(w)K \]
\[ P_C(w) = [q(l(w)) - wl(w)]K \]

The Hicksian logic, we have argued, maintains,

\[ V_U - P_U(w) = V_C - P_C(w) \]

Or

\[ w_{\text{MON}} l(w_{\text{MON}}) - wl(w) = q(l(w)) - [q(l(w)) - wl(w)] \]

This solves for \( w \).
The wage rate is less than the wage bill maximizing wage rate, $w_{MON}$, and greater than zero.

The wage rate is such that that the ‘concession’ the Confederation has given – the difference between the maximum profit rate and the actual profit rate – equals the concession Union has given – the difference between the maximum wages paid per unit of capital and the actual wages paid per unit of capital. So it may be described as a system of equal absolute concessions.

The solution wage rate has zero elasticity to equiproportionate increases in the supply of capital and labour. Thus the wage rate is invariant to any ‘balanced’ increase in capital and labour. Such balanced increases produce an equiproportionate increase in employment, and unemployment, but no change in the wage rate.

An increase in the supply of labour will reduce the wage rate. Employment will increase, but not by as much as the increase in labour supply. Thus a higher labour supply produces a lower wage, and higher unemployment.

An increase in the supply of capital will reduce the wage rate. Employment will also.

\[ q\left(\frac{\Sigma}{K}\right) + 2wl\left(w\right) - q\left(l\left(w\right)\right) = w_{MON} l\left(w_{MON}\right) \]

\[ q'\frac{d\Sigma}{K} + 2\frac{\partial W}{\partial w} dw - q' l' dw = 0, \quad W = wl \]

\[ \frac{dw}{d\Sigma} = \frac{q'}{K\left[q'l' - 2\frac{\partial W}{\partial w}\right]} < 0 \quad \text{and} \quad \frac{dl}{d\Sigma} = Kl'\frac{dw}{d\Sigma} = \frac{q'l'}{q'l' - 2\frac{\partial W}{\partial w}} < 1 \]

\[ q\left(\frac{\Sigma}{K}\right) + 2wl\left(w\right) - q\left(l\left(w\right)\right) = w_{MON} l\left(w_{MON}\right) \]

\[ -q\left(\frac{\Sigma}{K}\right) + 2\frac{\partial W}{\partial w} dw - q' l' dw = 0 \]
\[
\frac{dw}{dK} = \frac{q' \Sigma/K}{K[-q' l' + 2 \frac{\partial W}{\partial W}]} > 0 \quad \text{and} \quad \frac{dL}{dK} = l + K l' \frac{dw}{dK} = l + \frac{q' l' \Sigma/K}{-q' l' + 2 \frac{\partial W}{\partial W}} > 0
\]
‘The Strike Weapon’

Hicks did not present his model in terms of by struggle means of political weapons. Rather he supposed that the weapon each side used was economic: a refusal to supply the factor for production; ‘a strike’ in terms of the Union, and ‘a lockout’ the Confederation. This ‘boycotting’ of the production process can be considered as a ‘struggle resource’ that is free, or without direct cost. The ‘cost of victory’ in this struggle is solely an opportunity cost: the wage income forgone in the case of the Union, and the profits forgone in the case of the Confederation. The size of wage incomes and profit forgone obviously depends on the duration of the struggle.

Let $s$ the length of the strike/lockout, where the length of the period in question is unit. Thus the general expressions for the costs curve,

$$C_U = V_U - P_U(w)$$
$$C_C = V_C - P_C(w)$$

become

$$s_U w_{MOM} l(w_{MOM}) = w_{MOM} l(w_{MOM}) - wl(w)$$
$$s_C q\left(\frac{1}{K}\right) = q\left(\frac{1}{K}\right) - [q(l(w)) - wl(w)]$$

Thus far all might seem to follow closely the more general treatment of the previous section, but there is a special characteristic of the bargain when strikes and lockouts are the weapon. Since the struggle resource – boycotting the production process - is free is makes no sense to say ‘expenditure’ on struggle resource determines the victor. It is the quantity of this struggle resource that matters; the length of the strike/lockout; and in equilibrium each side believes there is some quantity of this resource – some length of stoppage – that will secure them victory. Since to both the Union and the Confederation have exactly the same opportunities to win – stoppages are free to

14 The closest military analogy is a siege: there is no battle, but whoever lasts the longest, wins the field
both– it seems sensible that in equilibrium both must have the same belief as to the stoppage that will secure victory Invoking this ‘symmetry’ we can write of just a single stoppage length,s

\[ s_U = s_C = s \]

This allows us to infer the solution is,

\[ \frac{wl(w)}{w_{\text{MON}} l(w_{\text{MON}})} = \frac{q(l(w)) - wl(w)}{\sum \frac{k}{K}} \]

It amounts to each party contracting from its ‘bliss point’ by the same proportion. Instead of an equilibrium of equal absolute concessions it is an equilibrium, of equal proportionate concessions.

A diagrammatic presentation

Following Hicks we can represent the equilibrium diagrammatically.

The concession curve of the Union is,

\[ s = 1 - \frac{wl(w)}{w_{\text{MON}} l(w_{\text{MON}})} \]

\[ -\frac{dw}{ds} = \frac{w_{\text{MON}} l(w_{\text{MON}})}{wl'(w) + l} \]

The concession curve of the Confederation is,

\[ s = 1 - \frac{q(l(w)) - wl(w)}{\sum \frac{k}{K}} \]
Figure 4 plots both concession curves, and identifies the wage outcome.

**Figure 4: The Stoppage Solution**

The solution when the wage is below the competitive wage

Figure 4 brings out that there seems no reason why the bargained wage may not be less than the competitive wage. And indeed it may. But the payoffs to Victory are different under Full Employment, so the solution needs to be revised again.

The value of peace terms to the Union if the peace term $w$ is below the competitive wage is $w \Sigma$. So the Union ‘concession curve’ becomes,

$$s = 1 - \frac{w \Sigma}{w_{MON} I(w_{MON})}$$
The value of peace terms to the Confederation if the peace term $w$ is below the competitive wage is $K_q \left( \frac{\Sigma}{K} \right) - w \Sigma$. So the Confederation’s ‘concession curve’ becomes,

$$s = 1 - \frac{K_q \left( \frac{\Sigma}{K} \right) - w \Sigma}{K_q \left( \frac{\Sigma}{K} \right)} ,$$

So the ‘strike weapon’ solution when the wage is below the competitive wage is,

$$\frac{q \left( \frac{\Sigma}{K} \right) - w \Sigma / K}{q \left( \frac{\Sigma}{K} \right)} = \frac{w \Sigma / K}{w_{\text{MHN}} l(w_{\text{MHN}})}$$

Figure 5: The Stoppage Weapon Wage May be Beneath the Competitive Wage
The equilibrium may be above the competitive wage rate (unemployment) or below the competitive wage (full employment).

**Comparative statics**

*An increase in K*

This will not shift the Union’s concession curves. Intuitively, this is because, for any given wage offered as peace terms, the payoff to Peace and the payoff to Victory are increased by the same proportion, so the cost of Victory (length of stoppage) that would leave it just worth Peace is unchanged.

But an increase on K will shift in the Confederation concession curve. Intuitively, this is because, for any given wage offered as peace terms, the payoff to Victory is increased by a lesser proportion than the payoff the Peace (This is because of diminishing marginal returns to capital: business gets the whole of output under Victory, but that is inelastic to the capital stock). The upshot is that the cost of Victory (length of stoppage) that would leave it just worth Peace is reduced.

The Confederation’s concession curve shifts left, and the wage rate rises.

*An increase in Σ*

This will not shift the Union’s concession curve. Intuitively, this is because, for any given wage offered as peace terms, the payoff to Peace is unaltered, and the payoff to Victory is unaltered. So the cost of Victory that would leave it just worth Peace is unchanged.

But an increase on K will shift in the Confederation concession curve. Intuitively, this is because, for any given wage offered as peace terms, the payoff the Peace is unaltered. But the payoff to Victory is increased. (Why? Because business gets the whole of output under Victory, and that increases with the size of the labour force).
So the cost of Victory that would leave it just worth Peace increases. So the curve shifts out and wage rate falls.

It’s not that workers are hungrier; rather it is capital that has become ‘hungrier’, as the payoff to them for victory is now greater. That is why the wage falls.

Benchmarking by some Axiomatic Bargain Solutions

Nash Bargain

How does the Hicks Bargain compare with the Nash Bargain?

Apply the Nash Bargain same situation previously analysed by the Hicks Bargain. A wage rate must be agreed upon by the Union and Confederation, if any employment is take place, implying, As before, decisions over the volume of employment remain the province of the individual capital owner. Thus both the wages bill and the profit bill map into \( w \); \( W = w (w) K \), \( \Pi = [q(l(w)) - w (w)] K \). Thus wage rate is the instrument by which the Union and Confederation bargain over the profit bill and the wages bill.

There is, of course, a relation between the wages bill and the profit bill, and it is not necessarily negative. The sign of the relation depends on the elasticity of labour demand.

\[
\frac{\partial W}{\partial \Pi} = e - 1
\]

If elasticity of demand exceeds 1 then an increase the wage rate increases both the profit bill and the wage bill, and is obviously in the interest of both parties.

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15 More precisely, a wage minimum must be agreed upon, but we will make that minimum as binding.

16 \( W = Kl(w)w \) and \( \Pi = Kq (l(w)) - w (w) K \)

\[
\frac{\partial W}{\partial w} = K [w' + l] \quad \text{and} \quad \frac{\partial \Pi}{\partial w} = K q' l - K [wl^{'+} l]
\]
Bargaining is plainly restricted to those contexts where interest diverge, which here is a situation of the inelastic demand for labour.

What point on the wages bill: profit bill trade-off is selected? The Nash solution is that which maximises

$$\max \ W\Pi$$

Thus

$$e - 1 = -\frac{W}{\Pi}$$

or

$$e(w) + \frac{1 - \pi(w)}{\pi(w)} = 1$$

This bears no resemblance to the equilibrium of the Hicks bargain. And the comparative-statics are different; in the Nash Bargain the wage has zero elasticity to $K$, and zero elasticity to $\Sigma$. The wage rate is rigid.

**Kalai-Smorodinsky**

Kalai-Smorodinsky solution (Upmann and Gerbr 2003) is that the outcome involves an equal proportionate shift from the maximal outcomes. This coincides with the outcome of the Hicks bargain when the strike is the weapon.

**Concluding Comment**

The paper has sought to rationalise Hicks’ theory, not to critically appraise it. But both the assumptions and the predictions of the present a rationalisation are open to criticism.

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$^{17}$ The theory supposes that for each side the length of the ‘boycott’ is an unrestricted choice variable. One could plausibly contend against this that strikers must eat, and so the size of their ‘strike funds’ puts a limit on the length of their strike. Similarly,
business must pay for fixed costs (debt charges) during a stoppage, and must draw on limited ‘liquid resources’ (such as stockpiles) to do so. This objection, admittedly, assumes that neither can borrow, but impediments to borrowing are also plausible.

The theory implausibly predicts that the wage share is less than the profit share

\[
\frac{W}{\Pi} = \frac{wl'(w)}{q(I(w)) - wl'(w)} = \frac{w_{MOS} I(w_{MOS})}{\frac{\sum q(\frac{\cdot}{K})}{K}} < 1
\]
References


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