A Note on A. C. Pigou’s Rejection of Pareto’s Law

by
Michael McLure¹
University of Western Australia
Business School – Economics Program

It is through induction that we know the form of the curve of income; and it is deduction that permits us to derive two very important theorems. The first of these theorems teaches us that the distribution of income is not due to chance. The second tells us that to increase the level of the minimum income or to reduce the inequality of income it is necessary that wealth increase more rapidly than population.

Vilfredo Pareto, 1897
*Cours d’Économie Politique*, vol. 2

1 Introduction

The secondary literature on ‘Pareto’s law’ is extensive² and historical studies, such as those by Joseph Persky (1992) and Terenzio Maccabelli (2009), have provided significant retrospective assessments of the debate pertaining to this law and its relationship to the question of income inequality. A. C. Pigou was one of the more influential contributors to the secondary literature with a full chapter in *Wealth and Welfare* (Pigou 1912), as well as a full chapter in each of the four editions of *The Economics of Welfare* (Pigou 1932), dedicated to rejecting the second theorem cited in the above quotation, which Pigou considers as ‘Pareto’s law’. In the historical literature, Pigou’s assessment is typically considered from the perspective of the critical insights that it brings to our

¹ I would like to thank Rebecca Doran-Wu for preparing the figures for insertion in this paper.
² A comprehensive sample of the English language secondary literature on Pareto’s law is included in volume 4 of *Vilfredo Pareto: Critical Assessments* (Wood and McLure 1999).
understanding of the limitations of Pareto’s law. But the rather vigorous mode of his largely flawed criticism of Pareto’s law suggests that his approach to this law is also relevant to the development of an understanding of Pigou’s general approach to welfare theory.

The purpose of this note is to consider Pigou’s motivation for writing his critical assessment of ‘Pareto’s law’. There are four objectives associated with that purpose. First, to overview the form of the Pareto distribution of income and graphically illustrate the intuition that underlies Pareto’s deduction of his second theorem (section 2). Second, to critically evaluate Pigou’s assessment of ‘Pareto’s law’ and, in the process, highlight the flaws in his analysis (section 3). Third, to consider Pigou’s motivation for rejecting Pareto’s law in terms of Pigou’s overall objectives for welfare economics and to suggest an alternate basis for criticising that law which is consistent with those objectives (section 4). Finally, to reflect on whether Pigou’s rejection of Pareto’s law of income distribution was partly motivated by his desire to apply the marginal theory of distribution, which Pareto had criticised, when developing the theory of welfare economics (section 5). The note ends with a brief conclusion (section 6).

2 Pareto’s Law

Pareto (1896, 1897) investigated whether the curve of distribution of income was associated with any particular form. For the levels of income above some arbitrarily determined minimum, he posited that income distribution may be represented by:
\( N_x = \frac{A}{(x + a)^\alpha} \) and \( \log N = \log A - \alpha \log(x + a) \)

\( N_x = \frac{A}{x^\alpha} \) and \( \log N = \log A - \alpha \log x \)

\( x \): a level of income
\( N_x \): number of people with an income of at least \( x \)
\( A \): parameter
\( a \): parameter
\( \alpha \): parameter

Based on his empirical tests of income tax data from many 19\textsuperscript{th} Century cities, regions and countries – such as England, Ireland, Italy, Prussia, Saxony, Peru, The Grand Dutchy of Oldenburg, Vaud, Paris, Florence (as well as data from 15\textsuperscript{th} and 16\textsuperscript{th} Century for some Swiss cities Basel and Augusta) – Pareto found that the form of the curve of distribution may be represented by the above equations as reasonable general approximations. As the parameter \( a \) was usually found to be very small, the more general
form of income distribution represented by equations 1\(^3\) or 1\(^*\) was sometimes set aside in favour of the simplified equations 2 or 2\(^*\). From the empirical results Pareto concludes that income is not distributed normally by pure chance, which is his first theorem. Instead:

(i) inequality is a general characteristic of the distribution of income;
(ii) the non-normal distributions equations 1 and 2 provide a reasonable approximation to the income distribution curve; and
(iii) the estimate value of \(\alpha\), the slope of the double log form of the Pareto distribution lies between 1.29 and 1.89\(^4\) although the average result was in the general vicinity of 1.5.

Pareto’s second theorem was derived from extensive deductive analysis, but such deductions were still undertaken in reference to his empirically estimated curve of income distribution. He posited that an index of equality in income distribution \(u_x\), which increases as inequality diminishes, may be represented by a quotient of the number of people \(N_x\) accruing income at least some arbitrary level \(x\), and, the number of people \(N_h\) accruing at least the minimum income \(h\). Given the form of the income distribution curve represented, Pareto measured income equality as quotients of equation 1: for the numerator, the reference value of income is \(x\); for the denominator, the reference value for income is \(h\). To simplify the analysis in this note, Pareto’s index of equality is represented by ratios of equation 2 (the reduced form of equation 1 in which parameter \(a\) is removed) and shown in equation 3 below.

\[
(3) \quad u_x = \left(\frac{N_x}{N_h}\right) = \frac{A}{x^\alpha} \cdot \frac{A}{h^\alpha} = \left(\frac{h}{x}\right)^\alpha
\]

\(^3\) Pareto introduced an even more general form of the income distribution \(N_x = \frac{A}{(x+a)^\beta} e^{-\beta x}\). But statistically the exponential term \(e^{-\beta x}\) was shown to approach 1 (because \(\beta\) was negligible in most cases), so that form was generally set aside.

\(^4\) Excludes estimates for 15\(^{th}\) and 16\(^{th}\) Century ‘Augusta’ and 18\(^{th}\) Century Peru, for which the data were considered ‘uncertain’.
Pareto (1897 [1971], pp. 998-999) derives his second theorem from his index of equality, that is, from equation 3, by considering the implications for income inequality associated with two specific cases:

Case 1: $\alpha$ is constant ($da = 0$) and minimum income ($h \neq 0$) is variable.

Equation 3 suggests that an increase in the minimum income $h$ is associated with a reduction in the extent of income inequality. As $da$ is zero in this case, Pareto concluded that the increase in minimum income can only be realised with a general increase in the aggregate value of taxable income when the number of taxpayers is unchanged, or, if mean taxpayer income ($z$) increases faster than the increase in the number of taxpayers. The result can be graphically represented in Figure 3 below.

To verify this result Pareto wrote the integral of the his income distribution curve (which, in the case of this illustration is equation 2 and not equation 1, as used by Pareto), which gives total taxable income by estimating the area under the income distribution curve from minimum income $h$ and some maximum income, and then derives equation 4.

$$ (4) \quad dz = \frac{\alpha}{\alpha - 1} dh $$

Case 2: $\alpha$ is variable ($da \neq 0$) and minimum income ($h = 0$) is constant.

To consider this case, Pareto expressed each side of equation 3 in logarithmic terms and demonstrated that the change in log $u_x$, which he labels $du_x/ux$, is related to the product of the logarithm of ratio ($h/x$) and the change in $\alpha$. The logarithm of the ratio $h/x$ is necessarily negative, because the minimum income $h$ is necessarily less that any income $x$. Consequently, an improvement in the equality of income distribution (a positive $du_x/ux$) is only possible when there is a reduction in $\alpha$, (a negative $da$) when $h$ and the taxpayer population are unchanged.
This case is simply illustrated in Figure 4 below, which shows that the increase in income equality due to a reduction in $\alpha$ is also associated with an increase in the area under the Pareto distribution, which indicates a growth in taxable incomes. When the numbers of taxpayers vary over time, the equivalent result is achieved when a reduction in $\alpha$ is associated with an increase in taxpayers’ mean taxable income (or real per capita income to grow when considering the entire economy).

In light of the results of these two cases, Pareto states his second theorem under which real per capita growth is a pre-requisite for an increase in the minimum income or a reduction in income inequality. But while Pareto argued that this means that “the problem of improving the conditions of the poor classes is above all a problem of the production of wealth” (Pareto 1897,[1971], p. 1097), Pigou (1912, p.72; 1932, p. 648) attempted to demonstrate that Pareto had failed to prove his case.
In Wealth and Welfare, Pigou commences his consideration of the topic by accepting that “the approximation to linear character in the income-curve [equation 2*] is, indeed, fairly well maintained, at least so far as incomes of moderate amount are concerned” (Pigou 1912, p.73), but he considered that the result was limited to the countries and cities examined by Pareto, and for the periods of time that Pareto investigated. That is, he appears to broadly accept that the Pareto distribution as a reasonable approximation for the cases investigated. However, he did not accept that the statistical evidence cited by Pareto, or the evidence of subsequent studies, supported the suggestion that $\alpha$ was constant in a temporal sense.

“The inclination of the curve, though it does not differ widely, still does differ distinctly in the different groups of statistics that have been observed.” (Pigou 1912, p.73)

In support of this conclusion, he cites Pareto’s own statistical estimates for $\alpha$ as well as those of other scholars like “Mr Bowley” and Rodolfo Benini. When considering the complimentary relationship that Pareto perceived between real per capita economic growth and the improvement of income equality, Pigou finds in Wealth and Welfare that Pareto’s statistical results are “not so entirely harmonious as he suggests” (1912, p.73). In The Economics of Welfare his language is much stronger, referring to ‘defects’ (1932, p. 649) in Pareto’s statistical interpretation and classing the basis of his statistical reasoning as ‘defective’ (1942, p.649).

This is the most surprising feature of Pigou’s analysis. Pareto’s second theorem of income distribution – the very law that Pigou sort to reject – was derived deductively from equation 3, which Pigou did not consider. Instead, he associates the result with $\alpha$ being constant over time and, in the process, completely ignores Pareto’s discussion of the relationship between the inequality of income distribution and variations in $\alpha$. As

---

5 In the equivalent section of The Economics of Welfare, Pigou writes: “it is true, the logarithmic income curve – at least for incomes of moderate size – is approximately a straight line” (1932, p. 649).

6 Pigou appears to reference equation 3 outside his chapter on Pareto’s law, but once again he misread what Pareto actually wrote when he observes that Pareto obtains his measure of inequality “by dividing the logarithm of the number of incomes above $x$ into the logarithm of $x$” (Pigou 1912, p.25).
such, Pigou appears not to have even recognised that the law he rejected was actually derived from analysis in which the slope of the curve of income distribution was reduced. So what did Pareto write in the *Cours d’Économie Politique* (1896-97 [1971]) that may have lead Pigou to believe that Pareto regarded $\alpha$ as invariant over time? There is no explicit text asserting that proposition, but perhaps the closest statement to that effect is contained in the summary at the end of the book:

“Experience has revealed to us a rather singular fact: the curve of the distribution of income varies little on average, in space or in time, in the civil populations for which we have given statistics.” (Pareto 1897 [1971], p. 1097)

Pigou may have read ‘varies little on average’ to mean that Pareto saw no scope at all for policy directed activities to alter income distribution. As Pigou no doubt understood that Pareto’s law had the potential to be directly and analytically derived from equation 2* if $\alpha$ is assumed to be constant, he may have just assumed that that was the relationship from which Pareto derived his law. That assumption may have been reinforced in Pigou’s mind by Pareto’s illustration of the income distribution curve using $\alpha=1.5$ (Pareto 1897 [1971], p. 987), which is a typical value for $\alpha$. But in commenting that ‘$\alpha$ varies little on average’, Pareto was certainly not suggesting that change in $\alpha$ was impossible. Rather, he was suggesting that scope for such change was constrained. Otherwise, Pareto would not have: highlighted the relationship between inequality and variation in $\alpha$ in his discussion of case 2, as discussed in the previous section; or provided a numeric example with particular values assigned to $h$, $z$ and $a$, that demonstrates how income inequality increases when the value of $\alpha$ increases from 1.5, in period 1, to 1.6, in period 2 (Pareto 1897 [1971], pp. 1001-1002).

In addition to questioning Pareto’s statistical evidence, Pigou also argued that social circumstances not evident in the range of countries and cities examined by Pareto, which were mainly 19th Century European cases, may have the potential to alter the form of the curve of income distribution and/or cause $\alpha$ to change. Chief among these were changes to inheritance laws and variations in the proportions of national dividend earned as wages and investment. In general, Pigou argued that society is made up of a number of ‘groups’, each of which are largely homogenous and the distribution of income within
each group is subject to normal Gaussian distribution laws and he then speculated that the non-normal distribution of income that Pareto observed for society as a whole was related to the different sizes and prevalence of the various ‘groups’ within the broader economy.\(^7\)

But Pigou asserts that Pareto’s statistical analysis covered a period in which the prevailing nineteenth century schemes of inheritance were generally maintained. From this, he insisted that a change in inheritance laws could indeed lead to a change in the form of income distribution. The implication of this objection is that changes to inheritance laws will lead \(\alpha\) to become more variable because the form of the distribution curve alters in response to a new assignment of property rights. He then criticises Pareto for failing to recognise in his *Cours* that empirical laws don’t necessarily continue to hold when arrangements related to the assignment of property rights change; and chided him for belatedly recognising this point in the *Manuale di Economia Politica* (1906 [2006]).\(^8\)

Along a similar line, Pigou speculated that the income distribution curve may alter its form in response to variations away from 19\(^{th}\) century European norms in the proportions of income that are ‘earned’ (from capacities for mental and/or manual work) and that which accrue through returns on investment.\(^9\)

In view of the points raised in the previous paragraph, Pigou concludes in *Wealth and Welfare* that “no ground would be given for believing that any given form of the income curve is necessary, in the sense that a cause altering one of these common circumstances would leave it unchanged.” (Pigou 1912, p.74). When his statistical concerns related to the temporal constancy of \(\alpha\) were combined with his recognition of

\(^7\) But Pigou’s observations on this issue are not original. He essentially restates, and embellishes upon, what Pareto had wrote in paragraphs 11 to 15, Chapter 9, of the *Manuale di Economia Politica* (1906 [2006], pp. 273-274).

\(^8\) Pigou’s summary of the cautious note in the *Manuale* is correct, but he does ignore a similar, although less forcefully stated, warning in the *Cours*.

“Certainly one can never be too prudent when dealing with purely empirical laws. However, the consequences drawn from this law will always hold, at least in the case of peoples for whom we have seen it [statistically] verified” (Pareto 1897 [1971], p. 985 ).

\(^9\) He illustrates this matter with respect to three points: (i) income from property is more unevenly distributed than ‘earned’ income; (ii) the greater the importance of unequal distribution of investment income, then the greater the potential for inequality in the distribution of earned income; and (iii) the distribution of training (an investment of capital in people) may alter the distribution of earned income in a manner that is independent of income from other forms of investment. Consequently, Pigou thought that inequality may diminish in response to: an increase in the ratio of ‘earned income’ to investment income; or an increase in the ratio of investment spending on the training of workers relative to other forms of capital investment.
the potential for the introduction of new legal-economic arrangements in regard to property right and changes in the form in which income accrues, Pigou finding in *The Economics of Welfare* is unequivocal.

“even if the statistical basis of the ‘law’ were much securer than it is, the law would rarely enable us to assert that any contemplated change must leave the form of income distribution unaltered.” (Pigou 1932, p. 655)

4 Implications for Welfare Economics

Pigou’s rejection of Pareto’s law was not motivated by a desire to establish an alternative general form of the income distribution. Rather, it was motivated by a desire to acknowledge a limited degree of inconsistency in the fundamental criteria that he wanted to apply to welfare economics. In *Wealth and Welfare* (1912, p. 66) and *The Economics of Welfare* (1932, p.123) welfare depends on two fundamental criteria: the size of the national dividend; and the manner in which it is distributed among the community. Pigou considered issues related to each criterion in isolation and then discussed the extent to which there was, in his words, ‘harmony’ or ‘disharmony’ between them. His *a priori* starting point was that that ‘absolute harmony’ between the two criteria was not possible. The challenge of Pareto’s law was that it suggested that there was no ‘disharmony’ between these two welfare criteria, which explains why he concluded that:

“no general proposition to the effect that improvements in the quality and in the distribution of the dividend [national income] *necessarily* go together, can be successfully maintained” (Pigou 1932, p. 655)

Having rejected Pareto’s law, he was free to use the ‘laws of distribution’ to consider: the relationship between the national dividend and the equality of ‘marginal net products’ (and various hindrances to that equality); and the effect of transfers to the poor,

---

10 In *Wealth and Welfare*, he also includes a third dimension when suggesting that an improvement in welfare is associated with a diminution in the variability of the national dividend, especially that part accruing to the poor (1912, p. 66). However, this issue was absorbed with his study of *Industrial Fluctuations* (1927) and it did not survive through to the last edition of *Wealth and Welfare* as a welfare criterion.
assessed using marginalist application of largely utilitarian principles. In the case of ‘disharmony’ between his two welfare criteria, *Wealth and Welfare* countenances a trade-off between the level of the national dividend and transfers from the ‘relatively rich’ to the ‘relatively poor’ through the introduction of a ‘national minimum’. In the face of a contraction in the national dividend, a welfare gain is still possible as long as the marginal gain from the transfer of each pound to the poor equals (or exceeds) the welfare loss from that contraction (1912, p. 396). In effect, Pigou considered it possible to purchase a net welfare gain by obtaining an improved distribution of income (a gain in terms of his 2nd welfare criterion) through a reduction in the national dividend (a loss in terms of his 1st welfare criterion). For such a net welfare gain to be recognised, subjective interpersonal comparisons of utility is necessary when applying the law of diminishing marginal utility to the incomes of different individuals.

While, Pigou’s motivation for rejecting Pareto’s law is evident when his interest in ‘disharmony’, that interest does not explain why much of his attack on that law was mis-specified, not just in terms Pareto’s own analysis, but also in terms of his own objectives for criticising Pareto’s law. In that regard, (i) the temporal constancy of $\alpha$ was not a claim that Pareto made; and (ii) discrediting the temporal constancy of $\alpha$ does not allow Pigou to illustrate the type of redistribution that he contemplated in *Wealth and Welfare* i.e. it does not demonstrate the possibility of income transfers from the relatively rich to the relatively poor achieved via the introduction of an *ex-post* nation minimum. To achieve that goal he needed to contest the proposition that the logarithmic form of the income distribution curve is accurately represented by a single straight line of slope $\alpha$. However, for the 19th century cities and countries considered in Pareto’s analysis, Pigou explicitly accepted the straight line log form of the distribution as accurate and when commenting on questions relating to the inequality of income distribution, he did so in terms of the slope of income distribution represented by Pareto’s curve.

“it is obvious, however, that a smaller slope of the curve [equation 2*] means a greater equality of distribution” (Pigou 1912, p.73).

But a reduction in the slope of the income distribution curve will only be associated with a contraction in national income unless if: (i) the reduction in the slope of
the income curve is confined to lower incomes; while (ii) the slope of the curve for higher
incomes earners increases. Otherwise, real per capita income will increase, as Pareto had
correctly demonstrated in the case 2. In light of his objectives for welfare economics,
rather than focusing on the supposed temporal consistency of $\alpha$, Pigou needed to sustain
his rejection of Pareto’s law by demonstrating the potential for $\alpha$ to change from a
constant value across all income levels, as per equations 1 and 2, to a curve that has at
least two distinct values for $\alpha$ that depend on income levels, one slope for people accruing
low income and another slope, or other slopes, for people accruing high incomes. In
specific relation to the double log version of the income distribution curve, Pigou needed
to demonstrate that the single straight line that Pareto that had estimated empirically for
all income levels can, following the introduction of a new redistribution policy, be
modified to form a kinked curve with at least two slopes, as shown in Figures 5 and 6
below.

![Figure 5: Log Distribution Curve - 1 kink](image)

![Figure 6: Log Distribution Curve - 2 kinks](image)

If he had pointed to the possibility of such a ‘kinked’ income distinction curve,
especially if he had provided statistical evidence for such a curve in countries that had
implemented minimum wage policies, then his rejection of Pareto’s law would have been
secure because Pareto’s analysis based equation 3 is predicated on $\alpha$ being the same across low and high incomes.\textsuperscript{11}

But while Pigou did not discuss the possibility of changes in the slope of the distribution curve being different for different income levels, his discussion of inheritance, which actually builds issues that Pareto raised in the *Manuale*, and changes in the proportion of income forms, could have been stepping stones to the consideration of such issues (although it is doubtful whether 20\textsuperscript{th} century experience with inheritance laws and income proportions have led to a rejection of the Pareto distribution).\textsuperscript{12}

Notwithstanding the fact that Pigou’s rejection of Pareto’s law cannot be sustained in terms of the analytical issues that Pareto raised in support of that law, a critical point is that Pigou’s rejection of the law has very clear implications for the shape of his welfare economics. The ‘Pareto welfare criterion’, under which a collective welfare improvement is associated with a gain to at least one member of the collective without harming any other member of the collective, is not a necessary criterion for a welfare gain in Pigou’s system because he applied a ‘subjectively measured’ interpersonal compensation criterion to reconcile the efficiency and equality dimensions of welfare economics. That is, he makes room for the possibility of subjectively assessed utilitarian benefits from redistribution of income outweighing the welfare loss associated with an ‘objective’ contraction in economic activity.

This contrasts with the objective ‘compensation’ principle that Pareto introduced to welfare economics (Pareto 1894 [2008]), whereby valuations of individual gain or loss are objectively represented by a numeraire good (which represents an ordinal indicator of welfare provided marginal utility is assumed to be positive). Pareto’s law is potentially consistent with the Pareto welfare criterion,\textsuperscript{13} but Pareto himself did not think of the issue in that way because his curve of income distribution was considered with respect to the

\textsuperscript{11} In fairness to Pigou, it should be stressed that his observation that the straight line of equation 2 is a reasonable approximation is qualified by the statement “at least so far as incomes of moderate amount are concerned” (Pigou 1912, p.73). This may suggest that he had been thinking of issues related to Pareto’s law in terms of how they influence low incomes only, but it would not be possible to reject Pareto’s analysis in support of his second theorem if high income earners are excluded from consideration.

\textsuperscript{12} It would be interesting to consider whether improvements in statistical measures and econometrics have represented more of a challenge to the Pareto distribution than have institutional changes altering the actual pattern of income distribution.

\textsuperscript{13} John Chipman (1974) has formally discussed the relationship between Pareto’s law and welfare rankings.
dynamic forces of income distribution and not the static analysis (with time considered in a purely ‘virtual’ sense) with which Pareto investigated economic welfare issues using the ‘Pareto welfare criterion’. Indeed, he used his notion of the circulation of elites to illustrate the dynamic income gains and losses that individuals face within society:

The outer shape [of the curve of income distribution] varies little; the inner part, on the contrary, is in constant motion; there are individuals who rise to the upper regions, and others who instead fall down. (Pareto 1906 [2006], p. 275)

Nevertheless, Pareto’s theorems on the law of income distribution are clearly consistent with his welfare economics once his objective compensation criterion is invoked. Objective compensation is always possible when a rise in the mean income is associated with a diminution of income inequality, even when the dynamic economic (and political) processes related to both the income level and the distribution of income involve gains for some and losses for others. It is also relevant that Pareto’s discussion of redistribution in the very same article in which he introduced the objective compensation criterion, ‘Il Massimo di Utilità dato dalla Libera Concorenza’ (Pareto 1894 [2008]), addresses the case of efficient redistribution in which there is no contraction in economic activity, with the minister of production selecting coefficients of production that minimise cost and maximise economic output, whilst the minister for justice determines the fair distribution of income.

Consequently, Pareto’s law is: consistent with Pareto’s broad conception of welfare economics when the objective compensation criterion is invoked and redistribution is considered without ‘destroying wealth’; and inconsistent with the complete system of welfare economics that Pigou was endeavouring to construct. Pareto’s law of income distribution and the objective ‘compensation’ principle in welfare economics are complementary, but Pareto’s law contradicts Pigou’s conception of welfare economics because it excludes the possibility of ‘disharmony’ between the level of the national dividend and the distribution of that dividend, which Pigou sort to reconcile through a subjective compensation notion based on interpersonal comparison of utility, subject to diminishing marginal utility of income.
5 The Marginal Theory of Distribution and Pigou’s Rejection of Pareto’s Law

In his letter of 14 February 1916 to J. B. Clark on the theory of marginal productivity, Philip Wicksteed noted that he had abandoned that theory after reflecting on a comment by Pareto:

“I was and am extremely interested in your independent arrival at my old conclusion; and it would be interesting indeed if you were to re-habilitate it after I had abandoned it. But I fear it cannot be done. What upset me was a remark of Pareto’s … Pareto said that the variables not being independent vitiated the argument. I take that to mean that the factors can never (unless by accident) be combined economically in exactly the same proportions either in two businesses or in the expansion of one. There will always be conditions which affect the relations of the different factors to each other” (Wicksteed, cited in Dorfman 1964, p. 295).

Pareto’s concerns with the marginal productivity theory of distribution were outlined in the *Cours* and the *Manuale*, as well as in various articles, and the issue has been systematically reviewed by Henry Schultz (1929). Perhaps the main point that Pareto repeatedly makes is that using marginal principles to derive the coefficient of production assumes that inputs all are variable and independent, but he regarded them as neither necessarily variable, as production involves a combination of fixed and variable factors, nor necessarily independent, because contemporaneous change in the composition of more than one input needs to be considered. Pareto was also concerned about requiring production theory to depend on constant returns to scale (Chipman 2002, p4).\(^{14}\)

Once having rejected Pareto’s law, Pigou immediately raised the possibility of using distribution theory when considering distribution issues. In *Wealth and Welfare*, the final paragraph of the chapter entitled ‘Pareto’s law’ restates the possibility of ‘disharmony’, but then diminishes that possibility by shifting his attention to harmony:

\(^{14}\) However, it should also be recognised that there is some debate over Pareto’s solution to these problems, with Hans Neisser (1940) demonstrating that the approach that Pareto eventually developed is broadly consistent with the marginal approach that Walras introduced in his last edition of the *Elements* in 1900; which itself was based on the introduction of variable coefficients of production to the general equilibrium theory of production by Pareto (1894 [2008]).
“Although absolute harmony can in no way be demonstrated, good reason can be found for the view that a limited, but, none the less, very important, measure of harmony exists. To display this, it is necessary to undertake some brief discussion of ‘the laws of distribution’. The problem of distinguishing the extent to which causes favourable to the one are also favourable to the other calls, therefore, for direct attack. In turning towards it, we find ourselves confronted with the broad problem of distribution” (Pigou 1912, p77).

The very next chapter, entitled ‘Production and Distribution’, considers the ‘laws of distribution’ in a marginalist manner, with reference to concepts like the ‘law of diminishing returns to individual factors of production’.

What is the significance of Pigou using the last paragraph of his chapter on Pareto’s law to introduce the discussion of the marginal theory of distribution that follows in subsequent chapters? While Pigou did not reflect on Pareto’s discussion of the marginal theory of distribution, the question must be asked whether his rejection of Pareto’s law was, from a sociology of scientific knowledge perspective, and attempt to discredit all that Pareto had to say on the issue of distribution, be it empirical or theoretical (including his concerns over the marginal theory of distribution)? The answer appears to be no. Pigou’s misreading of Pareto’s analysis in defence of his law of distribution make it clear that he read Pareto in an ad hoc manner and without giving too much attention to detail. In light of that, it is likely that his rejection of Pareto’s law was solely motivated by a concern with Pareto’s second theorem on income distribution. Conversely, it is unlikely that he was motivated by a desire to exclude Pareto’s views on the marginal approach to production from welfare economics.

6 Conclusion

Pigou was successful in demonstrating that Pareto’s law cannot be sustained if it is justified on the bases of a constant $\alpha$ over time, because the statistical evidence does not support the view that $\alpha$ is temporally constant. That finding was correct. The problem for Pigou, however, was that Pareto did not justify the law on the presumption that $\alpha$ was constant over time. He considered it variable, although with only modest change considered.
Like Pareto, Pigou regarded a diminution of inequality as being associated with a reduction in \( \alpha \), but, unlike Pareto, he did not appreciate that this necessarily leads to an increase in real per capita income when the Pareto distribution holds and slope \( \alpha \) spans all levels of income. If Pigou had followed Pareto’s deductive analysis and reflected on that analysis in light of Pigou’s own goals for welfare economics, he could have criticised the resulting law by noting that equation 2* has the potential to ‘kink’ in response to redistribution policy and lead to a contraction in the national dividend. Of course, in the absence of statistics to support such a kink, the proposition would be speculative but it would nevertheless reflect the theory that Pigou adopts when discussing transfers and the introduction of a national minimum income. This is not to suggest that Pigou’s analysis of Pareto’s law should be completely dismissed. For example, his discussion of forces that may lead to weaken the estimate of \( \alpha \) by altering the form of the income distribution curve was potentially useful as he developed empirically testable propositions (although he saw the matter in conceptual, rather than empirical, terms and did not text the propositions himself).

But of greatest importance is what this discussion of Pareto’s law reveals about Pigou’s own approach to welfare economics. It demonstrates clearly that the potential for ‘disharmony’ between redistribution and the level of the national dividend figures as a significant (although certainly not dominant) issue in his thinking on welfare issues. It implied that the theory of welfare economics required a capacity to consider the trade-off between wealth, on the one hand, and welfare, on the other. Pareto, however, did not have the same objectives for welfare economics and the consistency between his law of income distribution and the criteria he used for the study of welfare is evident and significant.

Finally, it is unlikely that Pigou’s desire to apply the marginal theory of distribution within his welfare economics was at all related to his rather forceful rejection of Pareto’s law of income distribution. There is no evidence in Pigou’s writing that he was aware of Pareto’s concern with production theory. When this is considered in conjunction with his misreading of what Pareto actually wrote on the relationship between income inequality and real per capita growth, it may be concluded that Pigou read Pareto in an ad hoc manner and with limited care.
References


