

Keynes, Pigou and the Supply Side of the *General Theory*

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I Introduction

This paper seeks to examine the formal, technical and mathematical connections between Pigou's *supply-side* analysis in chapters 9 and 10 of Part II of his *The Theory of Unemployment* (TU) and chapters 20 and 10 of Keynes' *General Theory* (GT), as well as the appendix to chapter 19. I will also compare and contrast the mathematical approach taken by Keynes in his *Treatise on Money* (TM) with that of the GT. This paper does not deal with the question of when Keynes intellectually gained the fundamental insights that allowed him to discover the idea, or theory, of effective demand.

Littleboy and Mehta (1987, 1989) have presented a persuasive case that Keynes already had discovered the idea of his theory of effective demand, and presented it in an embryonic form, in the TM. What is missing from the TM is an explicit technical exposition, in a formal, mathematical model, which incorporates the microeconomic foundations of firm and consumer optimising behaviour. By adopting Pigou's approach from TU, Keynes thought that he had plugged this hole. What Keynes overlooked was that Pigou's strictly mathematical treatment without supporting diagrams, was much too difficult for the vast majority of 1930's economists to master. The result was that Pigou's Part II, especially chapters 9 and 10, was overlooked. Thus, Keynes's nearly identical supply-side analysis in chapter 20 was also overlooked. Keynes's comparison of his chapter 10 and 20 with Pigou's chapters in the appendix to chapter 19 of the GT was overlooked as well, due to the difficulty of the mathematics.

These facts explain Patinkin's rejection of the Littleboy-Mehta argument, since for Patinkin, intellectual discovery is identical to a purely, formalistic mechanical manipulation of a macroeconomic model based explicitly on consumer-firm optimisation theory. Patinkin, however, has overlooked the connection between Pigou's TU and Keynes's chapter 20 and chapter 19 appendix in the GT. Thus, for Patinkin and many others, chapter 3 of the GT becomes the main relevant evidence, along with Volumes 13, 14 and 29 of the CWJMK, about when Keynes "discovered" his basic theoretical conclusions. By conflating theory discovery with theory development, Patinkin overlooks the possibility that the TM and GT are closely connected.

In summary, this paper will decisively lend support to the Littleboy-Mehta position while finding little support for Patinkin's position. Keynes already knew *what* he wanted to say in 1930. It was not until the appearance of Pigou's mathematical approach in 1933 that Keynes realised *how* to present what he may already have clearly understood himself in 1930. The next three sections cover the supply-side models of Pigou and Keynes, the connection between Keynes's modeling in the TM and GT, and the implications that these results have for historians of economic thought.

II The Mathematics of Keynes and Pigou

Keynes' mathematical development of his one and two sector supply models is contained in chapter 20. First, Keynes presents the micro foundations at the level of the firm-industry, using the subscript r to denote firm or industry. Then, after assuming a unit of measurement in which output as a whole can be measured (Keynes, p. 285), Keynes presents his macroeconomic version of the micro model, aggregating over the consumption goods and investment goods (capital goods) industries and firms.

Keynes uses the following notation and functional relationships. Let w = the money wage, p = the price level, r = a particular firm or industry, N = amount of labor, O = output, D = effective demand, p_r = the price of output of a firm or industry, $O_r = \phi_r(N_r)$ = the production function of a firm or industry, $O = \phi(N)$ is the aggregate production function, $\phi'_r(N_r)O$ and $\phi'(N)O$ are the first derivatives of the firm and aggregate production functions, the law of diminishing marginal product (returns) requires both $\phi''_r(N_r)O$ and $\phi''(N)O$, $D_w = D/w$ = effective demand in the aggregate, in wage units, $D_{wr} = D_r/w_r$ = effective demand in wage units at the level of the firm/industry, $p_w = (p/w)$ = the expected price level in wage units at the aggregate level, $p_{wr} = (p_r/w_r)$ = equals the expected price of a firm/industry, in wage units. $P(P_w)$ = economic profit. Keynes, on p. 282, using the employment function version, states that $N = \sum N_r = \sum F_r(D_w) = F(D_w)$. Here r can equal either 1 or 2. On pp. 115-116 of the GT, Keynes denotes N_2 to equal investment goods employment. Thus, by definition, N_1 must equal consumption goods employment. Keynes' analysis, given his aggregation assumptions on pp. 281-282 of the GT, allows his equations $(1-e_{or})/e_{er}$ to stand for either consumption or capital goods employment. On p. 285, he aggregates overall r industries, obtaining $(1-e_o)/e_e$, a macro result.

It must also be recognized that $p_w = (p/w) = 1/(w/p) = 1/\phi'(N) = 1/MP_N$ = the marginal product of labor, N , and $\phi'(N)O$ (or $\phi'_r(N_r)O$).

On p. 283 of the GT, in footnotes 1 and 2, Keynes defines

$$D_{wr} = P_{wr} O_r \quad (1)$$

But $O_r = \phi_r(N_r)$ and $P_{wr} = 1/\phi'_r(N_r)$. Substituting in, we get

$$D_{wr} = P_{wr} O_r = \phi_r(N_r) / \phi'_r(N_r) = Z_{wr} \quad (2)$$

where Z_{wr} is the micro aggregate supply function for a firm/industry. Equation (2) states that D_{wr} , effective demand for the firm/industry, in wage units, is a point on the firm/industry aggregate supply function, Z_{wr} . On p. 285, and in exposition in ft. 1 on p. 285, Keynes aggregates over all r firms and industries. Instead of (1), you obtain

$$D_w = p_w O = \phi(N) / \phi'(N) = Z_w \quad (3)$$

Z_w is the macro aggregate supply function.

Pigou initially defines X as a variable to represent the amount of wage goods labor. He defines $O = F(X)$ as his production function. $F'(X)$ = the first derivative of the production function equals the marginal product of labor, MP_X . Further, $p_w = (p/w) = 1/(w/p) = 1/F'(X) = 1/MP_X$. Thus Pigou's two sector model of an aggregated economy is

$$x + y = \phi(X) = p_w O = F(X) / F'(X) \quad (4)$$

where y = non-wage goods employment and aggregate output is measured in terms of wage goods.

The above model is a two-sector model. On p. 102, Pigou redefines X to stand for the quantity of labor "in the aggregate" and defines $O = F(X)$ to be "the value, in terms of wage goods, of the aggregate real output or income". Thus, I = Income = $\psi(X) = pO$. Then, the money wage,

$$w = [I / F(X)] F'(X) \quad (5)$$

$$= [pO / F(X)] F'(X) \quad (6)$$

This can be rewritten, after a rearrangement of terms, as

$$P_w O = F(X) / F'(X) \quad (7)$$

Pigou now has a one-sector supply model which is identical to the one-sector supply model of Keynes. Note that Pigou's "real demand for labor in the aggregate" elasticity is now

$$E_r = F'(X) / XF''(X) \quad (8)$$

or what was earlier denoted as ϵ_{ta} .

This occurs because, in the one-sector version of his two-sector model, $\phi(X) = X$ and not $\phi(X) = x + y$. Given $\phi(X) = X$, then $\phi'(X) = 1$ and

$$(X)(\phi'(X)) / \phi(X) = (X)(1) / (X) = 1. \quad (9)$$

Pigou's one-sector model is

$$X = \phi(X) = p_w O = F(X) / F'(X). \quad (10)$$

Note that all of the mathematical results obtained by Pigou with his two-sector model are duplicated exactly in the one-sector version.

Given his redefinition of X , Pigou's $F(X) / F'(X)$ is mathematically, and economically from the supply side, identical to Keynes's employment function, since

$$N = F(D_w) = F(p_w O) = F(\phi(N) / \phi'(N)), \quad (11)$$

where N = total aggregate employment. See pp. 280-283 of the GT.

Given Pigou's redefinition of X as aggregate employment, Keynes' result

$$(1 - e_o) / e_e = -[N\phi''(N)] / [p_w \{\phi'(N)\}^2], \quad (12)$$

where we have summed over all r firms and industries, so as to obtain a macro version of Keynes' micro result (Keynes, p. 283; Brady, 1990), is mathematically equivalent to Pigou's E_r , the only real difference being a negative sign.¹

First, note that $p_w = [1 / \phi'(N)]$. Thus, $(1 - e_o) / e_e$ can be rewritten as

$$(1 - e_o) / e_e = [N\phi''(N)] / \phi(N), \quad (13)$$

Now substitute Pigou's X for Keynes' N in the right hand side and use $O = F(X)$ instead of $O = \phi(N)$. Then

$$(1 - e_o) / e_e = -[X F''(X)] / F'(X). \quad (14)$$

Now Pigou's $E_r = F'(X) / X F''(X)$. Thus

$$E_r = -[e_e / (1 - e_o)] \text{ or} \quad (15)$$

$$-(1 / E_r) = (1 - e_o) / e_e \quad (16)$$

The only real difference between Pigou and Keynes is their different notation. Keynes's production function is $O = \phi(N)$; Pigou's is $O = F(X)$. The mathematical and economic definitions are identical with one exception. Keynes assumes the existence of a unit in which to measure aggregate output while Pigou measures *all* output, be it wages goods or non-wage goods, in terms of an equivalent amount of wage goods in both his one- and two-sector models.

Thus, Keynes's $D_w = \phi(N) / \phi'(N) = p_w O = Z_w =$ Pigou's $\phi(X) = p_w O = F(X) / F'(X)$. (17)

Keynes's Z_w (or $D_w = Z_w$) and Pigou's $\phi(X)$ are identical for the one-sector versions of their models from the supply side.

The fact that Keynes correctly derived his employment function in the GT in chapter 20 in footnotes 1 and 2 on p. 283 and also compared and contrasted it with Pigou's two-sector version in the appendix to chapter 19 of the GT on p. 273, should finally put an end to any further controversy about his effective demand and employment-aggregate supply functions. See, for example, the criticisms made by Asimakopulos², (pp. 20-25, 55-56) in his analysis of his effective demand and aggregate supply functions, or Marris, 1991, chapter 4.

III The Economic Models of the GT and TM

We now deal with the question of the connection between the analytic models used in the GT and TM, as well as attempting to specify the particular point in time at which Keynes breaks with the TM approach and moves on to the GT.

Let me make it very clear from the onset that I am dealing with the question strictly from a mathematical or formal point of view. I intend to answer the questions, "What connection is there between a mathematical approach using 'fundamental equations', as in the TM, and a GT approach using elasticities?", and, "At what point in time did Keynes abandon the TM framework and formally decide to use an elasticities approach?"

We will concentrate on the second 1933 draft table of the GT (Keynes, 1979, pp. 68-73) since, according to Keynes, "it is substantially the same as the fundamental price equation in my treatise on money."

Keynes' model is a variant of the standard $P = TR - TC$ model, where P = economic profit, TR = total revenue = pO and TC = total cost = wN , where w = money wage, N = employment, O = output and p = price. Then

$$P = pO - wN. \quad (18)$$

This can be rewritten as

$$pO = wN + P. \quad (19)$$

Substituting in from (18), we obtain

$$pO = wN + (pO - wN). \quad (20)$$

Keynes divides this by O to get

$$p = [(wN)/O] + [(pO - wN)/O]. \quad (21)$$

Keynes defines $E = wN$ and $C + I = pO$, where E = total money income, C = the nominal value of consumption goods output and I = the nominal value of investment goods output. Substituting into (21), we obtain

$$p = E/O + [(C + I) - E]/O. \quad (22)$$

Keynes defines $E - C = S$ = savings. Substitute in and rearrange terms. We obtain

$$p = E/O + (I - S)/O \quad (23)$$

or

$$p = (N/O)w + [I - (E - C)]/O, \quad (24)$$

which is identical to what Keynes obtains (Keynes, 1979, p. 71).

Keynes then provides the microfoundations for his macroeconomic analysis. First, assume perfectly competitive firms (1979, p. 72). Then totally differentiate $pO = wN + P$. (Keynes actually used Q to stand for P , where Q = short run quasi rents, i.e. short run economic profit, since, in the long run, due to the exit and entry of firms, only normal profits will result). We get

$$pdO + OdP = NdW + w dN + dP, \quad (25)$$

where we use "d" and not delta as Keynes did. Keynes assumed constant returns, i.e. $AR = MR$. Then, $dP = O$ in (25). Further, $OdP = O = NdW$, because, for the firm, p and w are fixed. Only O and N can change. Then we obtain $pdO = w dN$ or

$$dO/dN = w/p. \quad (26)$$

Given the production function, output is a function of $f(N)$. Then dO/dN is the marginal product of labor and equals the real wage. Keynes has presented very carefully the standard necessary, first-order condition for a P - maximum at the level of the firm. This condition also holds in the aggregate if one sums over all such firms.

Now consider Keynes's approach in the GT. Keynes' modeling is done on pp. 114-117, 280-286 and 304-306. He then compared his model with Pigou's in the appendix to chapter 19. Keynes starts out with the standard model

$$P = pO - wN. \quad (27)$$

Then,

$$pO = P + wN. \quad (28)$$

Keynes then denotes $pO = D$, where D = effective demand. Thus

$$D = pO = P + wN. \quad (29)$$

Now divide through by the money wage, w . Keynes obtains

$$D_w = p_w O = P_w + N. \quad (30)$$

Keynes denotes Z_w , aggregate supply, as equal to $P_w + N$. Given that $O = \phi(N)$ and $p_w = 1/(w/p) = 1/\phi'(N) = 1/MP_n$, where MP = marginal product, we get

$$D_w = p_w O = \phi(N)/\phi'(N) = P_w + N. \quad (31)$$

Keynes defines his *employment function* as

$$N = F(D_w) \quad \text{or} \quad (32)$$

$$N = F(D_w) = F(p_w O) = F(\phi(N)/\phi'(N)) = F(P_w + N) = F(Z_w). \quad (33)$$

(See Brady, 1990)

The supply-side analysis of the GT uses the *same* standard model as the TM. In the TM, Keynes divides through by O while in the GT, Keynes divides through by w . It should also be mentioned that Keynes's model above is identical to Pigou's model (1933, pp. 102-103) which was $X = \phi(X) = P_w O = F(X) / F'(X) = K_w + N$. (34)

Under constant returns, K_w , and P_w , would equal O . The reader should note that in the GT, Keynes uses capital $P(P_w)$ to stand for economic profit (GT, p. 283).

Based on the above exposition, I argue that Keynes, after reading TU and carefully studying Pigou's theoretical Part II, especially chapters 9 and 10, decided to adopt the Pigouvian approach to the standard form, $P = TR - TC$, which entailed dividing through by w and not O . Thus, gross national product, in wage units, equals $GNP = p_w O = P_w + N =$ aggregate supply, in wage units and is the sum of all economic and normal profits, wages and salaries.

In both the TM and GT, Keynes works with an aggregated version of perfectly competitive input and output markets. However, in using Pigou's approach, the microeconomic foundations for the macroeconomic model are made explicit instead of only being implicit, as in the fundamental equations.

The time period during which Keynes *mentally* abandoned the fundamental equations approach occurred shortly after his exchange with Robertson over Part II of Pigou's book (Keynes, 1973a, 310-319; 1979, 26-31). Of course, there would then be a significant time lag during which Keynes reworked his model, so as to make it as close as possible to Pigou's from the supply side. That would explain why *none* of the mathematical analysis, presented in a final form as elasticities in the GT, appears in *any* of the draft copies. Keynes alone is responsible for the final form his analysis took.

Finally, it is time to exorcise the "technical incompetence" theme, based on a supposed remark made by Gerald Shove, as reported by J. Robinson (1964, p.79), and repeated, for example, by Leroy³ (1983, p. 398, ft. 1), Gerrard (1991, p.277) and O'Donnell (1992, p.772). F. Hahn, (1982, pp. x, xi; 1985, pp. 15, 16) continues this theme by claiming that Keynes "left many gaping holes in his Theory."

An examination of the Keynes-Shove correspondence (Keynes, 1973a, pp. 321-326, 1973b, p.1; 1979, pp. 33-34) reveals that Shove readily admitted that *he* was analytically incompetent. In discussing the highly technical microeconomic exposition in Part II of TU, Shove freely admits that:

"Also, as you know I have absolutely no mathematics, so that in any case my opinion would not be worth much. But, so far as it goes, here it is (I am not, of course, competent to judge whether A.C.P.'s mathematical expressions are justifiable, and you must excuse my own crudities and howlers in this respect.)" (Keynes, 1973a, p. 322).

This raises a disturbing question. Given Shove's honest and forthright description of his lack of technical abilities, as well as Shove's clear realisation that Keynes was engaging him in what, at the time, were discussions of an advanced technical nature over the theory of value, it is very difficult to believe that Shove would then turn around and claim that, "Keynes never took the twenty minutes necessary to learn the theory of value." In fact, all we have is J. Robinson's *claim* that Shove made this statement. It is just as likely that J. Robinson misinterpreted Shove's point. Since Shove died in 1947 there is no way of verifying the remark, which, however, seems to have "taken" in the economics profession as a description of Keynes' abilities.

IV Implications For the History of Economic Thought

Marris claims, in his section III, "The Mystery of the General Theory", that he had:

"...always been troubled by the microfoundations (or lack of them) in the goods market. In my opinion, there are two things badly wrong with the General Theory. The first ... is that although the assumption is never stated in plain language, there are crucial words that effectively imply perfect competition. The second, associated weakness is that in a similarly roundabout way, the book appears to be assuming the neoclassical cost curve ... the General Theory (again using

evasive language) does, as is indicated in the following quotations, imply the inevitability of some kind of built in counter cyclical movement [sic] real wages." (Marris, 1992, p. 1240). Marris then presents three quotes from chapters 3 and 20. He then continues:

"As we shall see below, there is good reason to believe that all of the above passages were heavily influenced, if not actually written or revised, by Kahn." (Marris, 1992, p. 1240).

This comment by Marris is dubious, since Keynes clearly accepted the assumptions of perfectly or purely competitive input and output markets on p. 5 of the GT. Similarly, Keynes assumed the usual definition of the short run, where at least one input is fixed. In that case, if you take as the fixed input the capital stock (Keynes, p. 17; Pigou, p. 39), the law of diminishing marginal product applies and you *must* get countercyclical movements in the short run in real wages. Keynes presents his microfoundations in the very chapter, chapter 20, where 2 of the 3 quotations listed by Marris appear!

Contrary to Marris, and also Schumpeter (1954, p. 1122), who first started claiming that Kahn was a co-author of the GT, Kahn had very little to do with chapters 3 and 20 of the GT. I have already demonstrated above that Keynes patterned his analysis after that of Pigou.

Marris then cites a letter from Kahn in which Kahn claims that "Maynard derived from me the idea of thinking in terms of the supply curves", (Marris, p. 1242). Contrary to Kahn', Keynes' analysis of supply curves and functions in the GT is based on chapters 9 and 10 of Part II of TU.

Kahn's contribution to the GT was the employment multiplier contained in chapter 10 of the GT. Keynes later used this employment multiplier in a brief special two-sector version of his chapter-20 analysis. It is presented on pp. 273-275 of the GT. Keynes also thanked Kahn in his preface for his comments on earlier drafts. However, such acknowledgments do not imply co-authorship. Given the mathematical demonstration above, Kahn's claims have little support. However, see footnote 4 above.

Further support for my conclusion comes from Kahn's own analysis. Kahn states "Malinvaud includes another element in his story that is incompatible both with Walras and Keynes - an aggregate marginal cost curve, based, in effect, on the assumption of perfect competition." (Kahn, 1977. In Eatwell and Milgate, (ed.), 1983, p. 219).

He continues that

"Evidently, Malinvaud assumes a rising supply curve determined by marginal costs as in an old-fashioned textbook." (Kahn, op. cit., p. 219).

The footnote appended states

"There was an element of 'rising supply price' in Keynes's aggregate supply function, but it is not essential to the main argument. It was criticised on empirical grounds as soon as the General Theory was published." (Kahn, 1983, p. 217).

Kahn⁵ has overlooked the analytic structure of Keynes' GT, which, like Pigou's TU, is based on an analysis of an aggregated perfectly (or purely) competitive world. Keynes' goal was to show that even in such a "perfect" world, there can be involuntary unemployment and hence underemployment equilibria. All that is required is that the marginal propensity to consume be less than 1, or if one is including investment expenditure, that the marginal propensity to *spend* is less than 1. Then $Y_w \langle D_w = Z_w$, where Y_w equals actual total income expenditures and D_w is that particular level of expected effective demand corresponding to the necessary first order condition that the real wage equal the marginal product of labor. This D_w^0 is the result of solving an optimisation problem, give a maximum result, and specifies the maximum level of employment. Y_w is actual expenditure and represents what the employer actually receives. If this actual result becomes expected in the future, then an underemployment equilibrium will result. This result can occur if $C_w \langle D_{1w}$ and/ or if $I_w \langle D_{2w}$, where $C_w + I_w = Y_w$ and $D_{1w} + D_{2w} = D_w$.

We now can answer the question as to why Kahn (and J. Robinson) never involved themselves in the 45 year old aggregate supply function debate (see Patinkin, 1982, pp. 141-153; Brady, 1990). They overlooked, as did Friedman, Kahn and Asimakopulos, the formal model constructed in chapter 20 of the GT. Since this model is economically and mathematically identical from the

supply side to Pigou's one-sector model (see above), one can conclude that Kahn and Robinson overlooked the connection between Pigou's TU and Keynes's GT in their lifetime. It appears, from Marris' interpretation, that Kahn believed that Keynes was using an approach similar to that of his 1929 dissertation. Nowhere in any of the books or articles published by Kahn (or Robinson) in his (her) lifetime is there any mention of any connection between Pigou's TU and the formal mathematical approach involving elasticities used by Keynes in the GT.⁷

Patinkin has also written very recently on these issues. The first point to be made is that Patinkin, like Kahn and Marris, for example, has overlooked the connection between chapters 9 and 10 of Part II of Pigou's TU and Keynes' analysis in chapters 20, 21 and the appendix to chapter 19 of the GT.

Secondly, the claim that "... in another basic difference from the General Theory, the Treatise is devoid of marginal analysis" (p. 651) is inaccurate. Since the basic models of the TM and GT are identical, it would be more accurate to state that the marginal analysis is implicit in the TM and explicit in the GT. In both books, however, Keynes overestimated the mathematical competency of his contemporaries, who could not follow Pigou in Part II of his book.

Thirdly, the claim that T. Rymes' book "provides a unique and precisely dated record of the progress of Keynes' thought in the process of developing the General Theory" (p. 655) and the assertion that "... even before the publication of Rymes' book, the notes of Bryce and Tarshis were available to students of the chronology who realised that it was their scholarly responsibility to obtain them and take account of the evidence that they provided", (p. 655) is simply incorrect, since the Rymes' book (and the Bryce and Tarshis notes) is not *relevant* evidence. As I have demonstrated above, the turning point in Keynes' approach to writing the GT came after he had finished reading TU and corresponded with Robertson about it in September and October, 1933. Keynes then jettisoned the fundamental equations approach of dividing through the general model by O , holding output constant in the short run and adopted Pigou's approach of dividing through by w , holding money wages constant in the short run. This analytic change in Keynes' model appears nowhere in any of the above mentioned student notes.

And rightly so, given Keynes' comment on Pigou's book:

"Are the undergraduates to be expected to take it [Pigou's book] seriously? What a subject." (Keynes, 1973a, p. 313).

Given the advanced mathematical approach taken by Pigou and the fact that only Keynes himself had worked through the analysis completely, it is clear that such an analysis could not be presented to undergraduate students at Cambridge in the early 1930's. They simply did not have the necessary mathematical preparation.

Finally, Keynes' modeling of the supply side, at both the micro and macro level, is identical to the one-sector model of TU (pp. 102-103) and practically identical to the two-sector model of pp. 88-96 of TU. Yet in the correspondence between Patinkin and Kahn in October and November, 1978 (Patinkin, pp. 659-661) we find Kahn claiming responsibility for the analysis of the aggregate supply function (p. 659) and then stating that he "must confess that I am very puzzled in just the same way as you are" (p. 660) when confronted by Patinkin's claims that Keynes' analysis was "confused", "even erroneous", "at least at one point, incorrect", etc. (pp. 659-661).

Patinkin and Kahn are simply mistaken. The analysis of the aggregate supply function (employment function) in the GT is identical to the analysis presented by Pigou, although Pigou did not use Keynes' terminology. The only conclusion one can draw from this exchange of letters is that both Kahn and Patinkin have overlooked TU.

Contrary to Patinkin, Kahn's contribution to the GT is limited to the employment multiplier concept, which Keynes discussed on pp. 115-116 of the GT in terms of his aggregated one-sector model but later linked to a special two-sector model used in the appendix to chapter 19 of the GT in order to show Pigou what was missing from his supply side two-sector model. See pp. 272-279 of the GT.

Our second implication requires the following exposition from the GT. Keynes states on p. 44 that

$$p_r = Z_r / O_r = \phi_r(N_r) / \psi_r(N_r) \quad (35)$$

is the ordinary supply curve if $O_r = \psi_r(N_r)$, a production function. For notational simplicity, we will sum over all r firms and $r = 2$ industries and use $O = \psi(N)$. Then (35) reads as

$$p = Z / O = (N) / \psi(N). \quad (36)$$

Now Z , the aggregate supply function, equals wZ_w . Thus, $p = (wZ_w) / O$ but $Z_w = p_w O$ so $p = (wp_w O) / O = wp_w$. Thus, $p_w = 1 / (w / p) = 1 / \psi'(N) = 1 / MP_N = MC =$ marginal cost. Keynes' formula is that $P = MC = w / MP_N$ = the standard short run upward sloping supply curve of a perfectly competitive firm in the output market.

On p. 285, Keynes presents this analysis again, where $O_r = \phi_r(N_r)$ is the production function.

The only difference from p. 44 is that of notation. On this page, however, Keynes aggregates to obtain, in footnote 1, $p = p_w W$ (37)

$$\text{and } D = D_w W. \quad (38)$$

As above, $p_w = 1 / (w / p) = 1 / MP_N = 1 / \phi'(N)$. Thus, eq. (37) states that $p = p_w w = MC =$ the short run supply curve of a perfectly competitive firm in output space. Eq. (38) states that

$$w = D / D_w. \quad (39)$$

Now $D = pO$ and $D_w = p_w O$. Substituting in, we obtain $w = (pO) / (p_w O) = p / p_w$. But $1 / p_w = 1 / [1 / \phi'(N)] = \phi'(N) = MP_N$. Thus,

$$p / p_w = p \phi' N = VMP_N, \quad (40)$$

which is the value of the marginal product of N and is the standard demand curve for labor in input space.

We are now in a position to evaluate the claim of Milton Friedman (1974, pp. 148-149) that chapter 20 was "strictly peripheral" and "makes no contribution to the formal theory of the book", as well as his claim (1974, pp. 156-157) that Keynes' analysis in footnote 1 on p. 285 of the GT, which we have analysed above, consists of "definitions", "a truism" and "identities"⁸ which are "pure arithmetic and are completely empty both theoretically and empirically." Since the "identities" $D_w = p_w O$, $p = p_w w$ and $D = D_w w$ specify the supply curve in output space and the demand curve in input space for an aggregated perfectly or purely competitive economy, as well as specifying the necessary first order condition for a P-maximum and are identical to Pigou's results (Pigou, pp. 90-99, 102-103), Friedman's string of assertions can no longer be allowed to remain unchallenged. Otherwise, we would have to conclude that the perfectly (or purely) competitive theory of the firm is "completely empty both theoretically and empirically", which is false.

Contrary to Friedman, chapter 20 of the GT is an integral part of the formal theory and contains Keynes' microeconomic analysis, as well as the specification of his Employment and Effective Demand Functions.⁹ Friedman's assertion is equivalent to saying that Part II of Pigou's TU is "strictly peripheral" to the theory of TU.

V Conclusions

The standard, general, formal model used by Keynes in the TM and GT is identical to the model used by Pigou in TU. The specific presentation of particular alternate forms of this general model, $P = TR - TC$, varied depending on the exact purpose of the author. In the TM, Keynes concentrates strictly on a macro analysis, under the assumption of constant returns to scale, of real variables which might affect the price level, specifically the disequilibrium between I and S . Thus, Keynes divides through by O , de-emphasising his micro analysis.

By contrast, in the GT, Keynes seeks to make his microfoundations explicit in his theory of Effective Demand, as well as emphasising the role of output adjustment in determining the level of employment. Thus, he used, and improved upon, the Pigouvian technical approach of dividing through by a fixed money wage in a short period analysis of a perfectly or purely competitive economy with a fixed capital stock.

Chapter 20 contains the formal analysis of supply in the GT. For whatever reasons, this chapter has been overlooked by economists. In spite of the fact that $D_w = p_w O = \phi(N) / \phi'(N) = Z_w$, explicitly incorporates $p = MC$ pricing in the output market and the $w = VMP$ rule in the input market, many economists have been led to conclude that Keynes had no microeconomic foundations or that he was analytically incompetent. There is no mystery of the GT, if the mathematics of chapter 20 is worked out and compared with the mathematics of Pigou.

Once the connections between Keynes' GT and Pigou's TU analysis are understood the following conclusions about Keynes and his contemporaries in the history of economic thought result. First, there are no microeconomic or mathematical deficiencies in the GT. Second, the Keynes-Townsend exchanges have been severely misinterpreted. Third, Kahn and J. Robinson had little or nothing to do with Keynes's exposition of the supply side. Fourth, Keynes was the only economist of the 1930's who understood the technical developments presented by Pigou in Part II of TU. Fifth, chapter 20 of the GT, as well as the appendix to chapter 19, are very important parts of Keynes's formal theory. Sixth, Keynes directly responded to the criticisms made about the TM by explicitly incorporating optimising behaviour and clear cut, explicit, microeconomic foundations for his macroeconomic theory.

* 9426 Flower Street, Bellflower, California 90706, USA. The author thanks the three referees, two in particular, for helpful comments included in the text. He also thanks the editor for his encouraging comments. Any errors that remain are the author's.

Notes

- 1 A referee points out that the mathematical congruence of Keynes's chapter 20 and Pigou's chapters 9-10 of Part II of TU is due to Keynes' acceptance of the first classical postulate, and that this allowed Keynes to present the two theories in ways that allowed convenient comparison. I agree. However, this also means that Keynes's and Pigou's micro-foundations are the same and that close to 60 years of attacks on Keynes's supply-side analysis were, and are, unjustified.

Note also that, while Keynes and Pigou agree on the income (supply) side, they disagree completely on the expenditure (demand) side. Keynes's appendix to chapter 19 is a criticism of Pigou's failure to consider that expenditure is usually less than income. In making this point, Keynes also makes it clear, on p. 273 of the GT, that there are no real differences, other than notation, over the supply side.

- 2 If a reader of the GT concentrates primarily on chapter 3 of the GT, and overlooks the analysis contained in chapter 20, the appendix to chapter 19 of the GT, and chapters 9-10 of Part II of TU, then critical comments, for instance, like those of Asimakopulos, are justified relative to the partial evidence that is analysed. However, once the entire body of relevant evidence is examined, such critical comments lose much, if not all, of their force.

A referee asks how footnotes 1 and 2 on p. 283 of the GT translate into an aggregate supply function. Start with

$$D_w = P_w O = \phi(N) / \phi'(N).$$

Then

$$1 = [\phi(N) / \phi'(N)]. \quad [1 / D_w].$$

Let $1 = N/N$.

Then

$$N / N = [\phi(N) / \phi'(N)]. \quad [1 / D_w]$$

or

$$\begin{aligned} N &= [\phi(N) / \phi'(N)] \quad [N / D_w] \\ &= [O / \phi'(N)] \quad [wN / PO] \\ &= [O / \phi'(N)] \quad [N / O] \quad [w / P] \\ &= [O / \phi'(N)] \quad [N / O] \quad \phi'(N) \end{aligned}$$

$$\begin{aligned}
 &= [O / (w / p)] [N / O] \phi'(N) \\
 &= [pO / w] [N / O] \phi'(N) \\
 &= (D_w) (\phi'(N)) [N / O] \\
 &= [\phi'(N) / w\bar{O}] D, \text{ where } \bar{O} = O / N = \text{Average Product,}
 \end{aligned}$$

and

$N = F(D_w)$ is the employment function.

At an optimum, $D_w = Z_w$, so

$$N = F(D_w = Z_w) \text{ and}$$

$$N = F(Z_w) \text{ or } Z_w = \phi(N).$$

The aggregate supply function is just the inverse of the employment function. Rightly or wrongly, Keynes expected a technically trained reader to recognise this.

- A referee disagrees with my assessment of Leroy. I present the relevant statement and leave it to the reader to judge. Leroy states: "I am reminded by a referee of G. Shove's remark ... If the argument of this article is accepted, it is evidence that on the contrary, there were few or no economists of Keynes's generation who had a deeper intuitive understanding of price theory than Keynes. It is true, however, that Keynes was notoriously impatient with the mechanical details of elementary price theory."
- Only the underscored portions of Leroy's position are challenged by the author, given the analysis above.
- A referee believes that the following statement by Keynes lends support to the belief of Marris, Patinkin and Kahn that Kahn was the source for Keynes's aggregate supply function. "It was Mr. Kahn who first attacked the relation of the general level of prices to wages in the same way as that in which that of particular prices has always been handled, namely as a problem of demand and supply in the short period rather than as a result to be derived from monetary factors." I disagree and feel that, based on the exchange between Townsend and Keynes (Vol. 29, CWJMK, pp. 246-247), in which Keynes states that he was trying to get his algebra into much closer conformity with that of "others" than was actually the case, only one conclusion is possible. The "others" is actually one "other", A.C. Pigou. However, it is possible that the term "others" means Pigou and Kahn.
- In no way, shape or form am I commenting on Malinvaud's work or Kahn's commentary on Malinvaud. The only point that concerns us is what Kahn's comments on Malinvaud reveal about what Kahn's beliefs were concerning Keynes's handling of the supply side in the GT.
- A referee correctly points out that, *economically*, Keynes's D_w has a role assigned to it that has no counterpart in Pigou's book. The referee is correct. However, this involves linking Y_w , expenditure *realised*, to D_w , income *expected*. Under Say's Law, $Y_w = D_w (= Z_w)$. While my paper aims at dealing only with Keynes's supply side analysis, I will present the major theoretical difference between Keynes and Pigou. A much longer proof, which involves setting the elasticity e_p , from p. 116, ft. 2 of the GT equal to e_p on p. 283, ft. 2 and invoking Say's Law, $Y_w = D_w (= Z_w)$ is available from the author on request.

First, assume Say's Law holds. Then

$$C_w + I_w = Y_w = D_w = D_{w1} + D_{w2} = P_w O = \phi(N) / \phi'(N) = P_w N = Z_w.$$

But $C_w = bY_w$ and $I_w = (1-b)Y_w$, where $b = MPC$ and $1-b = MPI$. Then after substituting in, we obtain

$$bY_w + (1-b)Y_w = Y_w = D_w = \phi(N) / \phi'(N) = O / MP_N = (MPC)Y_w + (MPI)Y_w = O / MP_N = (MPC + MPI)Y_w = O / MP_N.$$

Then,

$$(MP_N)(MPC + MPI) = PO / Y_w = (w / p)(MPC + MPI) = O / Y_w.$$

Under Say's Law, Expenditure always equals Income. So $Y_w = D_w$. Substitute in $D_w = p_w O = p_w \phi(N)$.

We get

$$(w / p)(MPC + MPI) = \phi(N) / p_w \phi(N).$$

Cancelling the $\phi(N)$'s, we obtain

$$(w / p)(MPC + MPI) = 1 / p_w.$$

But $1 / p_w = MP_N$. Substitute in and we obtain

$$(w / p)(MPC + MPI) = MP_N \text{ or } (w / p) = MP_N / (MPC + MPI).$$

Other results are that

$$w = (p \cdot MP_N) / (MPC + MPI) = VMP_N / (MPC + MPI)$$

and

$$p = (w / MP_N) (MPC + MPI) = MC / (MPC + MPI).$$

Pigou's neoclassical results all implicitly assume $MPC = 1$ or $MPC + MPI = 1$.

Since Keynes's results are more general, they incorporate neoclassical theory as a special case of Keynes's General Theory.

7 Gram and Walsh state that

"It must never be forgotten that, as a member of the little group (known as the Circus) who worked with Keynes on manuscripts of *The General Theory* at Cambridge, she is in an excellent position to know what his ideas really were." (Gram and Walsh, 1983, p. 522, ft. 6).

J. Robinson states that

"You might almost say that I am the archetypal left-wing Keynesian. I was drawing pinkish rather than bluish conclusions from the *General Theory* long before it was published. (I was in the privileged position of being one of a group of friends who worked with Keynes while it was being written.)" (Gram and Walsh, p. 548).

The problem is that those early conclusions were based on an approach, the fundamental equations, which Keynes totally abandoned, in order to use the basic Pigouvian approach contained in chapters 9 and 10 of Part II of TU in the GT. I believe this occurred after Keynes' exchange with Robertson over Part II of TU in September-October, 1933. Keynes never shared his analytic modeling approach with anyone.

Thus, the conclusions J. Robinson was drawing "long before it was published" were only sound for those parts of the GT that Keynes let Robinson and Kahn read in his drafts. Of course, chapters 2, 3, 20, 21 and the allegedly troublesome mathematical analysis on pp. 44-45, 55-56, ft. 2, p. 116, ft. 1, and p. 126, ft. 1 in the GT only appeared in 1936 when the GT was published. But these are precisely the parts of the GT that serve as the formal, theoretical platform from which Keynes launched his theoretical attack on TU.

In her letter of June 16, 1935, J. Robinson, commenting on Keynes' draft copy critique of Pigou in the appendix to chapter 19 of the GT, states:

"This sounds very convincing but I never got far enough with Pigou to be able to offer any remarks. Throughout you want to alter marginal prime cost to supply price." (Keynes, 1973a, p. 645).

Of course, Keynes did not alter his wording, as it is correct. Of far greater importance, however, is J. Robinson's implicit admission that, as of June, 1935, she did not have the formal training in mathematics needed to understand the theoretical microeconomic analysis of Part II of TU (Pigou, pp. 33-108). J. Robinson, as with G. Shove, understood "the theory of value" in its graphical-diagrammatical form only. It follows, then, that she could not possibly have understood the formal mathematical analysis contained in chapters 20 and 21 of the GT, since, as I have already demonstrated, that analysis is practically identical to that contained in chapters 9 and 10 of Part II of TU. (Pigou, pp. 88-108).

8 If the results $D_w = p_w O$, $p = p_w w$ and $D = D_w w$ are taken in isolation from the rest of chapter 20 and the connection between Pigou's analysis on pp. 102-103 of TU is overlooked, then the conclusion, that they are definitional accounting identities, expressed in wage units, is a plausible conclusion. However, once the results of Pigou are compared with those of Keynes, I believe that such a conclusion has little support.

On p. 102, Pigou defines $l = pO$. For Keynes, $D = pO$. Pigou's $X = \text{Keynes' } N$. Pigou's $F(X) = \text{Keynes' } \phi(N)$. Pigou writes

$$w = [l / F(X)] F'(X),$$

which equals the value of the marginal product of X . For Keynes,

$$w = [D / \phi(N)] \phi'(N) = D / D_w, \text{ since}$$

$D_w = p_w O = \phi(N) / \phi'(N)$ and $1 / D_w = \phi'(N) / \phi(N)$. Thus, $w = D / D_w$ can be rewritten as $D = D_w w$, as in footnote 1, p. 285 of the GT.

Similarly, $p = p_w w$ can be derived. Since $l = pO$, $F(X) = O$ and $F'(X) = w / p$,

then $w = [pO / O][w / p] = p[w / p]$ or $p = w[p / w] = w p_w = MC = \text{marginal cost}$.

Pigou's derivations are carried out under the assumptions of perfect, pure, or free competition and profit maximization. Keynes defines capital $P(P_w)$ as profit on p. 283 of the GT.

In summary, from $D = D_w w$, one obtains $w = D / D_w = \text{the demand curve for labor in a purely or perfectly competitive input market}$. $p = p_w w = MC = \text{the supply curve in output space while } D_w = p_w O \text{ not only defines the effective demand for a expected price, but also the aggregate supply function indirectly, since}$

$$D_w = p_w O = \phi(N) / \phi'(N) = f(N)$$

which, assuming a maximum result, equals

$$P_w + N = Z_w = \phi(N).$$

The inverse function is

$$N = F(D_w) = F(p_w O) = F(\phi(N) / \phi'(N)) = F(P_w + N) = F(Z_w). \text{ See footnote 2 for the details.}$$

9 In an earlier draft of the GT, Keynes (1973, p. 452), in discussing constant returns to scale, states that this "would indicate that the employment function tends to approximate to a straight line drawn at a constant angle, which on the basis of the figures I have seen would be in the neighbourhood of 45°, between the axes of the quantity of employment and the effective demand measured in wage units, since there seems to be a nearly constant ratio (D/E) between dD and $dN \cdot w$, i.e. between dD_w and dN " where $D = pO$ and $E = wN$. Keynes used "delta" and not "d".

Of course, this discussion is identical to the GT result (1936, p. 283) that $dD_w/dN=1$. In footnote 2 on pp. 55-56 of the GT, given that $D_w=Z_w=\phi(N)$, if $\phi'(N)=1$, as Keynes states, we have another way of specifying constant returns, where Keynes's production function was expressed in the notation $O=\psi(N)$. Later, in his draft copy, Keynes provides yet another way (1973, p. 509) of expressing this, since " $p_w\phi'(N_r)=$ the marginal product of labor = the wage unit = 1". Summing over all r firms and industries, of which there are two, consumption goods and investment (capital) goods, we get $p_w\phi'(N)=1$, where Keynes now expresses his production function in a different notation, as $O=\phi(N)$.

Two additional ways of saying the same thing were likewise provided by Keynes. "If $\phi''(N_r)=0$, i.e. that the marginal productivity of labour is constant." (Keynes, 1973, p. 509). Likewise, "the condition $e_{or}=1$ means that $\phi''(N_r)=0$, i.e. that there are constant returns in response to increased employment". (Keynes, 1964, p. 284).

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