Misinterpreting Gustav Cassel

Origins and Implications for the Contemporary Literature

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This paper puts forward the proposition that Gustav Cassel, the modern originator of the purchasing power parity (PPP) hypothesis, has been misinterpreted and that the theory he suggested is not the theory that is portrayed in the contemporary literature. It is argued that this misinterpretation has led to several pitfalls, including the futile distinction between absolute and relative PPP and viewing PPP as an arbitrage relationship.

Introduction

It is very unfortunate for our "dismal science" that the contemporary literature, and the teaching material that is based on it, is littered with errors, misconceptions and pitfalls that can be attributed to the misinterpretation of some economists who came up with pioneering ideas some time ago. Nowadays there is a tendency by economists (applied economists in particular) not to read the history of economic thought and not to resort to the original writings, which can only give the misinterpretation legitimacy and acceptance within the profession. With the passage of time errors accumulate, the ideas based on the misinterpretation become established principles, and faulty practices flourish and become the norm.

Perhaps the most prominent manifestation of this somewhat provocative proposition is the misinterpretation of Gustav Cassel’s writings on the theory of purchasing power parity (PPP). This misinterpretation of Cassel, which was initiated by his contemporaries and accepted by subsequent economists, has led to the emergence of theoretically dubious propositions on PPP. By the 1980s, the empirical testing of PPP has become a "thriving industry", but the misinterpretation of Cassel, upon which the empirical testing is based, has also resulted in the emergence of inappropriate and invalid empirical testing practices.

This paper has two objectives. The first of these objectives is to demonstrate that Cassel has been misinterpreted on at least three counts. Specifically, Cassel has been misinterpreted with respect to (i) the PPP as an operational theory; (ii) the distinction between absolute and relative PPP; and (iii) the role of commodity arbitrage. The second objective is to illustrate the pitfalls in the contemporary literature on PPP, which have resulted from the misinterpretation of Cassel. The starting point is to demonstrate the misinterpretation of Cassel.
Misinterpreting Cassel I: PPP as an Operational Theory

Ever since Cassel formulated it as an operational theory early this century, the PPP hypothesis has taken various shapes and forms. The PPP theory has been portrayed as a theory of exchange rate determination, a theory of the transmission of world inflation, a short-run equilibrium condition, a long-run equilibrium condition, an arbitrage condition, a truism, etc. Notwithstanding the possibility that some of these representations may be valid, it is appalling that all of these shapes and forms have been incorrectly attributed to Cassel. The fact remains, however, that Cassel put forward the theory in one operational form only, although he allowed for an approximate but pragmatic representation that he used for the purpose of calculating post-war parities.

Before elaborating on what Cassel actually said, let us first consider how the contemporary literature portrays PPP. In his widely cited survey, Rogoff (1996, p. 647) defines PPP as "the disarmingly simple empirical proposition that, once converted to a common currency, national price levels should be equal". This implies the relationship

\[ P = SP^* \]  

where \( P \) is the domestic price level, \( P^* \) is the foreign price level and \( S \) is the exchange rate measured as the domestic currency price of one unit of the foreign currency. In another frequently cited survey, Taylor (1995, p. 19) states that (absolute) PPP "implies that the exchange rate is equal to the ratio of the two relevant national price levels". This implies the following specification:

\[ S = \frac{P}{P^*} \]  

which is equivalent to equation (1). Crownover et al. (1996, p. 783) also imply the specification represented by equation (2) since they define PPP as a "condition equating the level of the nominal exchange rate with the level of the price ratio". The same definition was used by Samuelson and Viner as the basis of their attack on Cassel. Samuelson's (1964, p. 147) critique of the theory was based on the definition that "PPP exists when the exchange rate equalizes the cost-of-living of the two countries."

The problem with these definitions and the resulting specification lies in words such as "equal", "equating" and "equalise", as well as the word "condition". Equations (1) and (2) represent a mechanistic relationship (a condition) that cannot hold, and which Cassel (1922, p. 183) described as a "pure dogma". This specification is preferred by economists attacking PPP on the grounds that: (i) it holds as a truism for any exchange rate under highly restrictive conditions (e.g. Chacholiades, 1990); (ii) it does not hold generally because it ignores other factors (e.g. Bresciani-Turroni, 1934); and (iii) it cannot hold by definition (e.g. Viner, 1937).

We start with an exposition of the first criticism. Let us assume (i) the absence of trade frictions such as transportation costs, tariffs and others; (ii) that countries use the same weights to construct aggregate price levels (not indices); (iii) the absence of non-traded goods; and (iv) that all goods are consumed in all countries since there is a perfectly free trade (as well as similar tastes and cultures!). Consider a two-country case, the home country and a foreign country. Assume that there are \( m \) domestic commodities with prices \( \pi_1, \pi_2, \ldots, \pi_m \) and \( n \) foreign commodities with prices \( \pi^*_1, \pi^*_2, \ldots, \pi^*_n \). If all of these commodities enter into the calculation of the domestic price level, \( P \), and the foreign price level, \( P^* \), then

\[ P = \sum_{i=1}^{m} \beta_i \pi_i + \sum_{i=1}^{n} \gamma_i \pi^*_i \]  

and
\[ P^* = \frac{1}{S} \sum_{i=1}^{n} \beta_i \pi_i + \sum_{i=1}^{n} \gamma_i \pi_i^* \] (4)

It follows that the price ratio is given by

\[ \frac{P}{P^*} = \left( \frac{\sum_{i=1}^{n} \beta_i \pi_i + S \sum_{i=1}^{n} \gamma_i \pi_i^*}{\sum_{i=1}^{n} \beta_i \pi_i + S \sum_{i=1}^{n} \gamma_i \pi_i^*} \right) = S \] (5)

which is equivalent to equation (2). This equation should hold as a truism, given the conditions on the basis of which it is derived. Since \( P \) is measured in domestic currency terms, \( P^* \) is measured in foreign currency terms and \( S \) is measured as the domestic currency price of the foreign currency, equation (2) is balanced with respect to the units of measurement. The equation also holds for any level of the exchange rate, and not only the equilibrium level. The violation of any of the underlying assumptions will cause a breakdown of this mechanistic equation.

We now illustrate the second criticism as expounded by Bresciani-Turroni (1934) who, according to Humphrey (1979, p 10), put forward the "most rigorous and systematic analysis of the PPP doctrine". Bresciani-Turroni presented a model determining the equilibrium level of the exchange rate, which may be written as

\[ S = \left[ \frac{Q}{Q^*} \right] \left[ \frac{1 + t}{1 + t^*} \right] R \left[ \frac{P}{P^*} \right] \] (6)

where \( Q \) is the quantity of exports, \( t \) is a measure of trade impediments, \( R \) is the ratio of the price of exports to the general price level, and an asterisk denotes the corresponding foreign variable. This equation implies that the equilibrium exchange rate is determined by variables other than the price ratio. Hence, equation (2) is misspecified.

The third criticism is based on the crucial difference between price levels and price indices. Even if the assumptions underlying equation (2) as derived in (3)-(5), were valid, the equation would not hold because in practice countries calculate price indices, not price levels. Price indices are measured relative to a base period, which means that they have no units. Equality cannot, by definition, hold between the exchange rate, which is measured in terms of two currencies, and the ratio of two price indices, which has no units. In his criticism of Cassel, Viner (1937) suggested that the ratio of price indices needs bear no relation to the exchange rate, arguing that PPP will not generally hold as a truism.

These criticisms are valid, but it can be demonstrated that Cassel never stated his theory in this manner, and that equation (2) is incorrectly attributed to him. Because of these (valid) criticisms, some economists modify the definition of PPP by avoiding words like "equal", resulting in specifications that are represented by functional relationships rather than equilibrium conditions. For example, O'Connell (1988, p1) defines PPP as "the simple proposition that national price levels should tend to be equal when expressed in a common currency". Lothian and Taylor (1996), on the other hand, define PPP as the condition which postulates that "the nominal exchange rate is proportional to a ratio of the foreign and domestic price levels".

These descriptions of PPP, which are also incorrectly attributed to Cassel, postulate that only one variable, the price ratio, determines the exchange rate. Viner (1937, p 383) criticises Cassel for "his insistence that the long run value of a currency
depends solely on the average level of prices in the two countries”. But again, this is not what Cassel said. So, what did he say?

Cassel’s theory is an operational theory that encapsulates the quantity theory of money and takes into account the effect of non-monetary factors on the exchange rate. At this stage it may be useful to say something about the theoretical origins of Cassel’s work, since the theoretical pedigree may shed some light on why Cassel would not have adopted the mechanical view of PPP. Cassel’s international fame rests on his contribution to the literature on monetary policy during and after World War I and on his book *Theoretische Sozialökonomie*, which was published in 1918. He started as a theorist with a paper entitled “Grundriss einer elementaren Preislehre”, which was published in *Zienschrift für die gesamte Staatswissenschaft* in 1899. In this paper he attempted to reformulate the Walrasian equations without using utility concepts. Cassel as a theorist was one of second generation economists who rounded off the Jevons-Menger-Walras structure. Moreover, Schumpeter (1961, p 862) argues that some of the opinions, in matters of pure theory, of modern Wicksellians may be still traced to his teaching. In short Cassel was an accomplished and a brilliant theorist. It seems to be rather difficult to digest the proposition that a theorist of his calibre could portray the PPP theory as it has been described so far.

Cassel’s theory postulates that (i) monetary factors (proxied by the price ratio) are the most important long-run determinants of the exchange rate; and (ii) other factors—such as trade impediments, transportation costs, capital flows and expectations—also play a role in the process of exchange rate determination. According to Gailliot (1970), Cassel’s theory amounts to saying that “in the long run important changes in the domestic price level have a much greater influence on the exchange rate than the real conditions of international trade”. Holmes (1967, p 694) argues that “the theory Cassel actually expounded contains several non-monetary variables and is presented within a framework which is conceptually similar to an econometric model in which the behavioral equations contain random terms”. Thus, Cassel’s PPP may be represented by the functional relationship

\[ S_t = f(P_t / P_t^*, X_t) \]  \hspace{1cm} (7)

where \( X \) is a vector of non-monetary variables that impinge upon the exchange rate. To Cassel, monetary changes were not the only factor determining the exchange rate. For example, he wrote the following to illustrate the role of trade impediments:

“The restrictions of which we have to take account in this connection may be of different kinds. Absolute prohibition of exports, prohibition with a system of licenses, rationing, export duties,..... are samples .... The restrictions can also take the shape of artificial or natural difficulties hampering transport” (Cassel, 1921, p 40).

In fact, Cassel made an attempt to estimate the effects of the increase in the U.S. tariff in 1922. He wrote:

“It is hardly possible to discern any definite effects on the exchange rate value of the dollar as a result of the considerable strengthening of American protectionism connected with the new tariff of 1922” (Cassel, 1928, p 270).

While Cassel regarded the price ratio as the fundamental factor determining the exchange rate, he referred to the secondary factors that cause deviations from the relationship, including trade impediments and capital flows. Cassel also wrote:

“As soon as an economic movement is a result of any several causes, it becomes the task of economic theory to explain the particular influence of each of these causes. In order to do so, it is generally necessary first to take out the most
important factor and study its separate effect which will then be represented by a
particular movement.... The art of economic theory to a great extent consists in the
ability to judge which of a number of different factors cooperating in a certain
movement ought to be regarded as the most important or essential one” (Cassel,

Hence, Cassel did not portray his theory as a condition of an equality between the
exchange rate and the price ratio (what he called the dogma), neither did he say that only
the price ratio affects the exchange rate. However, he did come up with a formula as a
pragmatic approximation to what happens under conditions of high inflation, or in the
long run. This is perhaps the reason for the misinterpretation of Cassel’s theory,
particularly that he used the words “rule” and “equal” to describe the relationship when
he wrote:

“When two currencies have been inflated, the new normal rate of exchange will
be equal to the old rate multiplied by the quotient between the degrees of inflation
of both countries. There will, of course, always be fluctuations from this new
normal rate, and in a period of transition these fluctuations are apt to be rather
wide. But the rate calculated in the way indicated must be regarded as the new
parity between the currencies” (Cassel, 1921, p 37).

This view of PPP postulates that the ratio of the equilibrium exchange rate in the current
period, \( S_t \), to the exchange rate in some base period, \( S_0 \), is equal to the ratio of the
domestic price index, \( P_t \), to the foreign price index, \( P^*_t \). Therefore, the PPP relationship
is represented by

\[
S_t = S_0 \left( \frac{P_t}{P^*_t} \right)
\]  

where the price indices are measured relative to the base period 0. This is only a rough
and pragmatic approximation to a more general relationship obtained by assuming that
only monetary factors change. In this respect, Cassel wrote:

“The arithmetical process reproduced above is therefore satisfactory for a first
rough calculation of the new equilibrium level rate of exchange after big
monetary changes have occurred.” (Cassel, 1932, p 661).

This is the description of the equation that Cassel used to determine the appropriate
levels of the exchange rates between major currencies after the end of World War 1.

Since \( P_t = 1 + \hat{P}_t \) and \( P^*_t = 1 + \hat{P}^*_t \), where \( \hat{P}_t \) and \( \hat{P}^*_t \) are the domestic and foreign
inflation rates respectively, it follows that

\[
S_t = S_0 \left[ \frac{1 + \hat{P}_t}{1 + \hat{P}^*_t} \right]
\]  

which means that the equation can be written either in terms of price indices or the
corresponding inflation rates.

We are now in a position to respond to the criticisms outlined earlier by showing that
Cassel did not state his theory as anything similar to what is represented by equation
(2). First, we have seen that the price ratio in Cassel’s theory is a proxy for relative
monetary conditions. If \( P/P^* \) reflects relative monetary conditions, then \( P \) and \( P^* \)
must represent the general price levels that are inevitably measured by some indices. This
specification lacks the logical consistency because, as Holmes (1967, p 687) succinctly
puts it, “the ratio of two arbitrarily weighted commodity price indexes will not equal to
any given number”. In this case, Holmes adds, “no empirical evidence is relevant in
determining the truth or falsity of the dogma”.
Second, Cassel did not use the word "equal" in his exposition of the theory except when he described the approximate formula. Throughout his writings, Cassel used qualifying phrases such as "determined by", "vary as", "broadly proportional to", "primarily determined by" and "tends to be". This description cannot be compatible with equation (2).

While the criticism of Breciani-Turroni is valid, it should not be directed at Cassel. Indeed, Cassel did more than Breciani-Turroni to demonstrate the effect of other factors on the exchange rate. While the Breciani-Turroni model (equation 6) identifies four determining factors for the equilibrium exchange rate (terms of trade, relative transportation costs and tariffs, relative prices, and the price ratio), Cassel recognised a broader set of factors, including expectations and speculation (see Humphrey, 1979). The only difference is that Cassel recognised the complexity of the process determining the equilibrium exchange rate and the measurement of the underlying variables, and opted to concentrate on the comparative statics of moving from one equilibrium exchange rate to another under the influence of monetary shocks while other factors were kept unchanged. This is a legitimate procedure. Holmes (1967, p 692) attributes Cassel's frequent use of qualifying phrases, such as "is proportional to", to his desire to "theorize about the relation of the exchange rate to purely monetarily caused price changes when all non-monetary variables were held constant".

As far as Viner's criticism is concerned, Cassel himself had pointed out many years earlier that PPP could not be viewed as a truism (or a quantitative formula as Viner put it). He wrote:

"The view that the theory is a self-evident truism is quite as superficial as that it is entirely wrong.... the theory is dependent upon certain assumptions, and as long as these assumptions are not filled, the actual rates of exchanges may deviate more or less considerably from those representing the purchasing power parity." (Cassel, 1928, p 25).

Viner's (1937) criticism of Cassel is not valid for the same reason as the criticism of Breciani-Turroni: Cassel never claimed that the price ratio is the only variable in the equation. Holmes (1967, p 690) finds it rather strange that Viner criticised Cassel on these grounds, in view of their public discussion in 1928 following Cassel's lecture at the Harris Foundation. In that lecture Cassel explicitly stated that shifts in demand or supply functions could cause deviations from PPP.

It is perhaps appropriate to conclude this section with the interesting remark of Balassa (1964, p 584) that "the PPP doctrine means different things to different people". Balassa was absolutely right, but for Cassel PPP meant one thing only, the operational theory that relates the nominal exchange rate to prices or inflation rates, and which can be approximated - if the other factors that may influence the exchange rate are of secondary importance - by the equation he used to calculate the post-war parities (equation 9).

Misinterpretation of Cassel II: The Distinction Between Absolute and Relative PPP

Dissatisfaction with the specification of absolute PPP as a relationship between the levels of the exchange rate and the price ratio (as represented by equation 2) has led economists to put forward the so called relative PPP hypothesis, postulating a relationship between changes in the exchange rate and inflation rates. Taylor (1995, p 19) defines relative PPP as the relationship that "changes in the exchange rate are equal to changes in relative prices". Similarly, Crownovwer et al. (1996, pp 783-784) define relative PPP as "a
condition equating changes in the nominal exchange rates to changes in the price ratio. Again, the distinction between absolute and relative PPP is wrongly attributed to Cassel.

The derivation of relative PPP is based on the assumption that absolute PPP holds in a base period and the current period (0 and t respectively). If \( S_0 = P_0 / P_0^* \) and \( S_t = P_t / P_t^* \), then

\[
\tilde{S}_t = \tilde{P}_t - \tilde{P}_t^*
\]

(10)

where \( \tilde{S}_t \) is the rate of change of the exchange rate. Because equation (10) is written in percentage terms, the problem of the logical consistency associated with equation (2) disappears. Relative PPP, as represented by equation (10), is normally put forward as a comparative statics proposition and commands greater acceptance by economists. Yet, it does not represent Cassel’s PPP.

Cassel’s contemporaries and subsequent economists wrongly attributed the distinction between absolute and relative PPP to him (e.g. Pigou, 1922; Terborgh, 1926; Balassa, 1964; Humphrey, 1979). Holmes (1967, p 694) argues that relative PPP “has been advanced by several economists but not by Gustav Cassel”. But Humphrey (1979, p 8), for example, attributes the distinction to Cassel on the basis of the following quotations: “the quotient between the general levels of prices in two countries.” (Cassel, 1916, p 62) and “the old rate multiplied by the quotient of the degree of inflation”. (Cassel, 1922, p 140). Humphrey interpreted the first quotation to imply absolute PPP and the second to imply relative PPP. This interpretation is rather odd, since the “quotient between the general levels of prices” and the “quotient of the degree of inflation” lead to the same result as far as Cassel’s equation is concerned. We have already shown that Cassel’s equation can be written either in terms of price levels (equation 8) or inflation rates (equation 9).

Since \( S_t / S_0 = 1 + \tilde{S}_t \), a simplification of equation (9) gives us the equation

\[
\tilde{S}_t = \tilde{P}_t - \tilde{P}_t^*
\]

which is identical to relative PPP (equation 10). While this may appear, on the surface, to be the case, the two equations have different stories behind them. Relative PPP is based on the dogmatic PPP, requiring \( S_t = P_t / P_t^* \) for all \( t \), and thus not allowing for the effect of other factors. Cassel’s equation, on the other hand, is a rough approximation of a general functional relationship (equation 7). Cassel had in mind one version of PPP, which he did not label “absolute” or “relative”.

Misinterpretation of Cassel III: The Role of Commodity Arbitrage

The contemporary literature views PPP as a generalisation of the so called law of one price (LOP). This “law” can be represented by

\[
\pi_i = S \pi_i^*
\]

(11)

where \( \pi_i \) and \( \pi_i^* \) are the domestic and foreign prices of commodity \( i \). Equation (11) represents a relationship that is maintained by commodity arbitrage. To move from the LOP to PPP, we have to make the (heroic) assumption that commodity arbitrage enforces price parity across a sufficient (meaning wide) range of individual commodities. This leads to

\[
\sum \pi_i = S \sum \pi_i^*
\]

(12)

If the weights used to construct price levels or indices are similar across countries, then we obtain equation (1).

While economists derive equation (1) as an arbitrage relationship, they wrongly attribute the underlying line of reasoning to Cassel. For example, MacDonald (1995)
derives PPP from the LOP but he refers to it as Cassel's view of PPP. This is not so, because Cassel's PPP is simply an extension of the quantity theory of money to the case of an open economy (Moosa, 1996).

The monetary view of PPP is obvious in Cassel's writings. Cassel (1916) explained his theory by viewing the exchange rate to be "an expression for the value in the money of one country put upon the money of another country" such that "the rate of exchange between the two countries will be determined by the quotient of the general levels of prices in two countries". Since the quantity theory of money stipulates that the general level of prices varies, other things being equal, in direct proportion to the quantity of money - or what Cassel called the "circulating medium" - it follows that "the rate of exchange between the two countries must vary as the quotient between the quantities of their respective circulating media" (Cassel, 1916, p 62). Cassel's theory is in fact an extension of the quantity theory of money to the case of an open economy, in which commodity arbitrage is not stated as the transmission mechanism.

There is indeed a big problem with the arbitrage view of PPP. It has become evident that there is little evidence for the ability of commodity arbitrage to enforce convergence of prices measured in the same currency (see Rogoff, 1996). This is because of transportation costs, non-traded goods, trade impediments and differing national standards (e.g. voltage). Because of these factors, Rogoff (1996) concludes that "outside a fairly small range of various homogenous goods, short-run international arbitrage has only a limited effect on equating international goods markets prices". Rogoff extrapolates this conclusion to PPP by saying that "given the abject failure of the law of one price in microeconomic data, it is a little wonder that tests based on aggregate price indices overwhelmingly reject purchasing power parity as a short-run relationship". Similarly, Dutton and Strauss (1997) attribute deviations from PPP to the presence of non-traded goods, implying that commodity arbitrage is the force maintaining equilibrium.

If the arbitrage view of PPP were valid, then how could we explain the evidence that PPP works well under hyperinflation? Is it that commodity arbitrage works only under hyperinflation? Was it arbitrage that led to the working of PPP during the German hyperinflation as the evidence provided by Frenkel (1978) shows? Obviously, this line of reasoning does not make any sense. There must be another transmission mechanism whereby the effect of monetary expansion is transmitted to the exchange rate via prices. We do not need to be concerned here about what this transmission mechanism may be, but beyond a shadow of doubt it is not commodity arbitrage.

Now that the misinterpretation of Cassel has been (hopefully) made clear, it is time to examine the pitfalls that can be found in the contemporary literature. These pitfalls, it is argued, have resulted from the misinterpretation of Cassel.

Pitfall I: The Distinction Between Absolute and Relative PPP for Empirical Testing

Frenkel (1978, 1981) seems to have set the rule by testing absolute and relative PPP using specifications in levels and first differences respectively. These specifications are written as

$$ s_i = \alpha_0 + \alpha_1 p_i + \alpha_2 p_i^* + v_i $$

(13)

and

$$ \Delta s_i = \beta_0 + \beta_1 \Delta p_i + \beta_2 \Delta p_i^* + u_i $$

(14)

where $s_i$ is the logarithm of the nominal exchange rate and $p_i (p_i^*)$ is the logarithm of the domestic (foreign) price level. Alternatively, the equations can be rewritten by imposing the symmetry restriction as
\[ s_i = \alpha_0 + \alpha_i r_i + u_i \]  
and  
\[ \Delta s_i = \beta_0 + \beta_i \Delta r_i + u_i, \]

where \( r_i = p_i - p_i^* \). The specifications given by equations (13)-(16) are evident in the bulk of the published econometric work including, inter alia, Giovannetti (1989), MacDonald (1988), Corbae and Ouliaris (1988), Patel (1990), Conejo and Shields (1993), Serletis (1994), and Mahdavi and Zhou (1994). Corbae and Ouliaris (1988) note that tests for cointegration of relative PPP cannot be conducted because inflation rates and the first differences in exchange rates are stationary. Patel (1990) asserts that it is difficult to test relative PPP because observed exchange rates and prices are difference stationary, incorrectly implying that relative PPP must take a first difference form. On the other hand, Conejo and Shields (1993) and Mahdavi and Zhou (1994) wrongly classify their models into those representing absolute PPP and others which represent relative PPP on the basis of the order of integration of exchange rates and prices. The argument goes as follows. If \( s_i \sim I(1) \), \( p_i \sim I(1) \) and \( p_i^* \sim I(1) \) then absolute PPP is tested on the basis of equation (13). If, on the other hand, \( s_i \sim I(2) \), \( p_i \sim I(2) \) and \( p_i^* \sim I(2) \), then relative PPP is tested on the basis of equation (14). This argument implies that absolute PPP is valid for some countries while relative PPP is valid for others (hyperinflation countries)! The (embarrassing) question that arises out of this faulty line of reasoning is the following: what version of PPP it would be if, for example, \( s_i \sim I(1) \), \( p_i \sim I(2) \) and \( p_i^* \sim I(2) \)?

Junge (1984) was the first economist to draw attention to the confusion between absolute and relative PPP by arguing that if \( p \) and \( p^* \) were price indices, then equation (13) would not represent absolute PPP because standard types of price indices are computed relative to some base period. Thus, both equations (13) and (14) represent relative PPP in levels and first differences respectively. This point, however, has been largely overlooked in the subsequent literature.

Thus, while absolute PPP describes the relationship between exchange rates and price levels, relative PPP describes the relationship between exchange rates and price indices. Since comparable data on price levels are unavailable, absolute PPP is not testable empirically, while relative PPP is. And as long as empirical testing is based on price indices (which is inevitably the case), it will amount to testing relative PPP irrespective of the model specification. Therefore, the distinction between absolute and relative PPP is empirically redundant or useless. After all, it seems that economists would be better off testing PPP in levels without calling it absolute or relative.

There is an exception: Crownover et al. (1996) did test PPP using price levels. However, their results only verified Pippenger's (1993) argument that the use of comparable data on price levels, if available, is likely to produce results that may not differ significantly from those obtained by testing relative PPP using price indices. Ironically, Crownover et al (1996) used the price levels also to test relative PPP, and so they legitimised the absolute-relative distinction. We shall later show that even if we overlook the difference between price levels and price indices, this distinction does not make any sense.

**Pitfall II: Testing PPP in First Differences**

In this section it is shown that there are hazards in testing PPP on the basis of the first difference model. The first hazard is that if the exchange rate is cointegrated with prices, then the first difference model is misspecified. That is, if PPP as represented by equation
(13) or (15) is a valid long-run relationship, then equations (14) and (16) are not valid representations of the short-run dynamics. This is because, according to Granger’s Representation Theorem, cointegration implies the existence of a valid error correction model. Hence, equations (14) and (16) should contain an error correction term in order to be correctly specified.

The second hazard is more serious. For the purpose of illustrating this hazard, a distinction is made between conventional PPP and ex ante PPP. Conventional PPP is written in terms of the actual values of the variables as represented by equations (13)-(16). Ex ante PPP, on the other hand, is written in terms of the expected values of the first differences. The argument put forward here is that testing the first-difference PPP model amounts to testing ex ante PPP and, therefore, the empirical validity of the first-difference model indicates the failure, rather than the validity, of (conventional) PPP.

The first-difference and ex ante PPP models are given respectively by

\[ \Delta s_t = \Delta p_t - \Delta p^*_t \]  
(17)

and

\[ \Delta s^*_t = \Delta p^*_t - \Delta p^*_t \]  
(18)

Clearly, the only difference between first-difference PPP and ex ante PPP is that while the former is backward looking, the latter is forward looking approach. Ex ante PPP predicts that the change in the real exchange rate derived from the rationally-expected changes in nominal exchange rates and relative prices is a zero-mean serially-uncorrelated process \( (e_t = \Delta s_{t+1} - \Delta p_{t+1} - \Delta p^*_t) \).\(^6\) Therefore, testing PPP in first differences amounts to testing ex ante PPP, which means that results indicating white noise residuals in the model should imply the failure rather than the validity of conventional PPP. Thus, the first-difference PPP equation does not represent a correct test of PPP. This conclusion exposes the pitfalls of interpreting the results obtained by a number of economists - including, \textit{inter alia}, Frenkel (1978), Manzur (1990, 1991), Conejo and Shields (1993) and Mahdavi and Zhou (1994) - who were led to conclude by testing the first-difference model that PPP was not rejected.

\textbf{Pitfall III: The Distinction Between Absolute and Relative PPP as a Theoretical Proposition}

In this section we question the validity of the distinction between absolute and relative PPP from a theoretical (as opposed to an empirical) perspective, ignoring for this purpose the difference between price levels and price indices. To start with, one cannot help wondering why no other relationship (such as the consumption function or the quantity theory of money) is classified into absolute and relative versions by considering it at a particular point or over a period of time respectively. Is it not the case that when economists write the consumption function as a linear relationship between consumption and income, they are essentially interested in the marginal propensity to consume, which is a measure of the \textit{change} in consumption in response to a unit \textit{change} in income? Why do not they instead write the “absolute consumption function” in levels and the “relative consumption function” in first differences?\(^7\)

Let us consider the PPP relationship that is represented by the linear equation

\[ S_t = \alpha + \beta R_t \]  
(19)

where \( R \) is the price ratio. Equation (19) can be used to find the value of \( S \) for any value of \( R \) (at a particular point in time); it also gives the percentage change in \( S \) (between two
points in time) corresponding to the percentage change in \( R \) during the same time
interval. For two points in time, 0 and 1, the following must hold
\[ \frac{S_i - S_0}{R_i - R_0} = \frac{S_1 - S_0}{R_1 - R_0} = \frac{\Delta S}{\Delta R} \]  
Therefore
\[ S_i = S_0 + \frac{\Delta S}{\Delta R} (R_i - R_0) \]  
Equation (21) contains \( S \) and \( R \) in both levels and first differences. So, while
equation (19) looks, on the surface, to be a representation of absolute PPP, it can be
rewritten to encompass terms that reflect both absolute and relative PPP. The fact of the
matter is that when two variables are related by a functional relationship like (19) it does
not make any sense to distinguish between the relationship as written in levels and first
differences, and create two versions of the same relationship on the basis of this
distinction.

**Pitfall IV: Testing Coefficient Restrictions**

If equation (2) represents PPP, then equation (13) must satisfy the symmetry and
proportionality conditions, requiring \((\alpha_0, \alpha_1, \alpha_2) = (0, 1, -1)\). In equation (15) the
restriction \((\alpha_0, \alpha_1) = (0, 1)\). Economists have tried to test these restrictions by using
improper methods.

The problem is that if \( s \) and \( r \) in equation (15) are nonstationary and cointegrated,
the OLS estimates of the coefficients will be superconsistent but not fully efficient.
Moreover, their standard errors will not be asymptotically normal, and so it will be
erroneous to derive inference on the basis of the conventional \( t \) statistics. A number of
economists either derived inference, or implied the possibility of doing so, on the basis of
the conventional standard errors and \( t \) statistics. Heitger (1987), for example, wrongly
concluded that the coefficient on the price ratio was not significantly different from one
on the basis of the standard \( t \) statistics. Phylaktis (1992) reached the conclusion that the
property of proportionality did not hold, just by looking at the numerical value of the
cointegrating parameter which was estimated by the Johansen procedure. Another
example is presented by Sarantis and Stewart (1993) who tried to derive inference in the
same way, but qualified their attempt by stipulating that the magnitudes and signs of the
coefficients on the price ratio were not “particularly meaningful” in the absence of
cointegration. The fact of the matter is that even if cointegration is present, deriving
inference on the basis of conventional standard errors is erroneous. This is because
cointegration guarantees the superconsistency but not the efficiency of the estimated
coefficients.

Only a small number of economists tested the coefficient restrictions properly.
Cheung and Lai (1993), for example, used the procedure proposed by Johansen (1988),
while Ardeni and Lubian (1989) tested the restrictions on the basis of the procedure
suggested by West (1988) to correct the \( t \) statistics and make them asymptotically normal
(see also Moosa, 1994). However, other procedures that can be used for this purpose
have been largely ignored in the PPP literature. These include the Engle and Yoo (1991)
three-step method and the Phillips-Hansen (1990) fully modified OLS procedure can also
be used for this purpose.

What is important to remember here is that while it is essential to test the
significance of the estimated coefficients properly, one should not expect the conditions
of symmetry and proportionality to hold. In Cassel’s theory, proportionality is possible
only under hyperinflation, when monetary factors dominate, and possibly in the very
long run. In Cassel’s theory, not even exclusiveness should be expected to hold, because factors other than prices affect the exchange rate.

Summary and Conclusion

This paper has put forward the argument that Gustav Cassel, the modern originator of the purchasing power parity hypothesis, has been misinterpreted and that this misinterpretation has led to several pitfalls in the contemporary literature. The following are the various facets of the misinterpretation of Cassel:

1. While Cassel presented an operational theory, in which the exchange rate is affected by monetary and non-monetary factors, his theory is portrayed as a mechanistic equality between the price level and the price ratio.
2. It is claimed that Cassel suggested the distinction between absolute and relative PPP. However, no such distinction is evident in his writings.
3. It is also claimed that Cassel viewed PPP as an arbitrage relationship, when Cassel’s theory is actually an extension of the quantity theory of money to the case of an open economy.

It was also argued that these misinterpretations have led to several pitfalls, and it was demonstrated that:

1. The distinction between absolute and relative PPP is redundant for the purpose of empirical testing because price indices are invariably used for this purpose. However, the underlying specifications can still be useful for differentiating between static and dynamic representations of PPP.
2. Testing PPP in first differences is faulty because if PPP is valid then the first difference model is misspecified, and because favourable results imply the failure of PPP.
3. The distinction between absolute and relative PPP as a theoretical proposition is useless.
4. Economists have used faulty procedures to test the coefficient restrictions implied by the properties of proportionality and symmetry.

Clearly, the misinterpretation of Cassel has led to some testing problems. There is now a wide body of literature dealing with the empirical testing of the PPP hypothesis. These studies employ a variety of testing techniques ranging from the conventional OLS method to TVP estimation and spectral analysis. Yet, there is no consensus view on the empirical validity of the hypothesis. One reason for this is that efforts have been concentrating on the testing techniques rather than on a re-examination of the theoretical construct. So, when economists believing in the validity of the hypothesis obtain negative results, they attribute the failure of PPP to “the low power of the test”. It is perhaps the case that a re-specification of the model in line with what Cassel was trying to say would be a more appropriate line of research.

It is rather unfortunate then that the literature on PPP, which is one of the most extensively studied hypotheses in economics at large, is littered with these pitfalls. One can only hope that researchers would be more careful by not taking everything written on PPP for granted. It is also hoped that this paper has done Cassel, a great economic theorist, some justice.
I am grateful to two anonymous referees for their comments on an earlier version of this paper. I would also like to thank Michael Schneider for providing some insights into the origins of Cassel’s work and for bringing my attention to some important pieces of work on the history of economic thought.

Notes

1 There are few exceptions, however. For example, Crownover et al. (1996) test PPP by using data on price levels prepared by the German Statistical Office, which are used primarily to adjust the salaries of German diplomats and foreign service personnel stationed outside Germany. Gilbert and Kravis (1954) developed price level measures for common baskets of goods across the U.S., U.K., France, Germany and Italy. More recently, Summers and Heston (1991) constructed estimates covering a much broader range of years and countries.

2 Indeed, Cassel’s thinking on monetary theory falls within the Wicksellian heritage as portrayed in Wicksell’s classic Geldzins und Guterpreise, which dealt with the determination of the price level in an economic system (see, for example, Leijonhufvud, 1997). For this reason, Cassel adopted the monetary view of PPP, which is unlikely to have produced the mechanical view of PPP.

3 Although Cassel did not include a statistical error term in his equation, he discussed random fluctuations literally.

4 In the modern textbook treatment of PPP, equations similar to equation (7) are utilised to illustrate deviations from, or the failure of, PPP in terms of other factors. However, what is overlooked is that Cassel himself stressed the importance of these factors over eighty years ago.

5 Only few studies recognise the fact that equations (13) and (15) represent relative rather than absolute PPP (see, for example, Beach et al., 1992; and Moosa, 1994). On the other hand, see MacDonald (1993) and Cooper (1994).

6 For the derivation, see Bhatti and Moosa (1994, p 149).

7 It is noteworthy that one of the virtues of cointegration analysis is that it has provided a rule of thumb for specifying models in levels and first differences. Traditionally, models were specified either arbitrarily or on the basis of a theoretical hypothesis. The advent of cointegration analysis alone should have provided a good incentive for abandoning the absolute-relative dichotomy.

8 In this respect, a more legitimate distinction can be made between statics, comparative statics and dynamics.

References


Beach, E.D., Cottrell, N.H. and Uri, N.D. (1992) Relative purchasing power parity under fixed and flexible exchange rates, Economia Internazionale, 45, 4-32.


