

MARSHALL ON BALANCED GROWTH FROM THE SUPPLY SIDE:  
A NOTE

Anthony Y.C. Koo and Warren J. Samuels\*

The concept of balanced growth has been given a variety of meanings. It has been viewed occasionally from the demand side and sometimes from the supply side. The purpose of this note is to call attention to a discussion in Alfred Marshall's Principles which, while Marshall did not intend consciously to develop the idea (and is therefore only an inadvertent precursor), nonetheless contains the basic ingredient of the concept of balanced growth from the supply side.<sup>1</sup> His analysis is essentially neoclassical in that it contemplates the market by itself generating balanced growth. The note also will formalize Marshall's model both graphically and algebraically. It turns out that the Marshallian idea can be readily expressed in a linear programming model if we assume for simplicity a fixed proportion of factor combination.

The discussion comes in Book IV, on the agents of production, in Chapter X, the third of the five chapters on industrial organisation, this dealing with the concentration of specialized industries in particular localities. After examining the advantages of localized industries, Marshall notes the disadvantages of a too extensive demand for one kind of labor and the possible remedy in the growth of subsidiary trades:

In those iron districts in which there are no textile or other factories to give employment to women and children, wages are high and the cost of labour dear to the employer, while the average money earnings of each family are low. But the remedy for this evil is found in the growth in the same neighbourhood of industries of a supplementary character. Thus textile industries are constantly found congregated in the neighbourhood of mining and engineering industries, in some cases having been attracted by almost imperceptible steps; in others, as for instance at Barrow, having been started deliberately on a large scale in order to give a variety of employment in a place where previously there had been but little demand for the work of women and children.

Accordingly, he argues,

The advantages of variety of employment are combined with those of localized industries in some of our manufacturing towns, and this is a chief cause of their continued growth.

In addition,

A district which is dependent chiefly on one industry is liable to extreme depression, in case of a falling-off in the demand for its produce, or a failure in the supply of the raw material which it uses. This evil again is in a great measure avoided by those large towns or large industrial districts in which several distinct industries are strongly developed. <sup>2</sup>

Clearly, these paragraphs contain the basic idea of balanced industrial growth from the supply side.

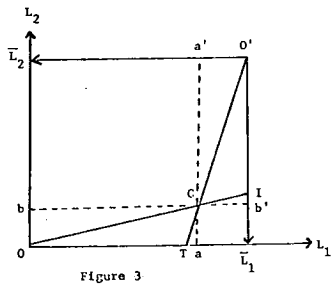
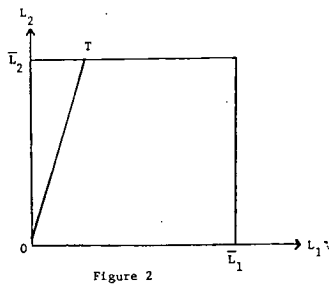
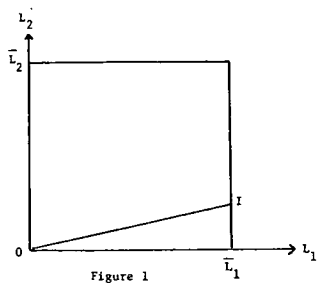
The discussion is of interest in several other respects. First, the discussion indicates Marshall's appreciation of the Smithian idea of a wide extent of the (labor) market through diversification as a complement to concomitant concentration of specialized industries in particular localities. Second, the development of both localized industries and subsidiary trades, and therefore a complex labor market, in the circumstances of what now would be called balanced growth from the supply side, comports with, and indeed is an example of, a key Marshallian point, the enlargement of interdependence among economic agents. Third, Marshall's phrase "by almost imperceptible steps" is a manifest example of his overriding principle of continuity, of the incremental nature of change. Fourth, by noting instances in which large-scale industries have been started deliberately ... in order to give a variety of employment" (as distinct, say, from arguing that they were started to take advantage of available labor), Marshall seems to be suggesting the existence in the labor market of a phenomenon which he later notes in commodity markets, namely, instances in which a monopolist is "desirous to promote the interests of consumers",<sup>3</sup> that is, in effect, the inclusion of the interests of

workers and consumers, respectively, in the utility function of a manufacturer-employer. Finally, it seems clear that Marshall's analysis is much deeper than a naive adherence to Say's Law. Absent diversification and the growth of a market for subsidiary labor, there is no reason to presume full employment.

Marshall's pregnant but homely if not discursive use of the idea of balanced growth may be more precisely developed.

Suppose that only iron is produced in an economy which has two factors, say male and female labourers ( $L_1$  and  $L_2$ ). Total resources of the economy are given ( $\bar{L}_1$  and  $\bar{L}_2$ ). Assume also that only one process can be used. The situation is depicted in Figure 1.  $L_1$  is drawn along the X-axis and  $L_2$ , the Y-axis. OI is the output expansion path along which the combinations of  $L_1$  and  $L_2$  in fixed proportions will be used to produce iron. Figure 2 presents the story of the textile industry where more  $L_2$  relative to  $L_1$  will be used for each unit of textile than in the case of iron as evidenced by the difference in the slope of OI and O'T. Maximum economic growth demands, and balanced growth would consist of fuller utilization of resources by an appropriate combination of the levels of output of both of these industries - although there is no presumption that maximizing income (value of output or social welfare) requires full employment of all factors.<sup>4</sup> The equilibrium division of resources between the industries can be arrived at graphically. We turn figure 2 by 180° and superimpose it on figure 1, as shown in Figure 3. Oa of  $L_2$  will go into the iron industry and O'a' of  $L_1$  and O'b' of  $L_2$ , the textile industry. If there is only iron industry in the economy, a part of  $L_2$  will be unemployed (IO'). Similarly OT of  $L_1$  will be unemployed if textile be the only industry.

Algebraically, the Marshallian model can be formulated in several alternative ways. Let  $A_{ij}$  represent the total  $i^{\text{th}}$  factor ( $L_1$  and  $L_2$ ) needed to produce one unit of  $j^{\text{th}}$  good (I for iron and T for textile). We have:



$$A_{11}I + A_{12}T \leq \bar{L}_1 \quad (1)$$

$$A_{21}I + A_{22}T \leq \bar{L}_1$$

$$I \geq 0, T \geq 0 \quad (2)$$

One way of looking at the maximizing process is to say: (a) the economy maximizes the net output  $I$  ( $T$ ) subject to a prescribed net output of  $T(I)$  and (1) and (2), (b). By introducing prices of iron and textiles  $p_i$  and  $p_t$ , we may restate the above and say that the economy maximizes the national product  $p_i I + p_t T$  subject to (1) and (2), (c). On the assumption of the existence of a community utility function,  $U(I, T)$ , we may choose to maximize the utility function subject to (1) and (2) by forming the corresponding Lagrangian multiplier of  $U(I, T) + \lambda_1 (\bar{L}_1 - A_{11}I - A_{12}T) + \lambda_2 (\bar{L}_2 - A_{21}I - A_{22}T)$ . The dual to the programming models can also be added as other possible ways of viewing the problem.

One may argue, however, that the formulation up to this point is static in the sense of achieving the maximum output of welfare. Where is the growth path? The answer can be given each time in terms of demand expansion of either industry. Consider a rise in demand for iron. The expansion of the iron industry calls for an inflow of male labor which in turn results in an inflow of female labor. An expansion of textile production will have a similar effect, although the ratio of increase of male and female likely have a similar effect, although the ratio of increase of male and female likely will not be the same. Whatever the relative proportions, an expansion of one will induce the expansion of the other. conversely, the aggregate income and employment in an economy with a diversified economic base will be more stable (on the assumption that disturbances affecting different industries are not completely correlated). Knowledge of this stability tends to encourage investment, especially by risk averters. By having larger values (for expansion) for  $\bar{L}_1$  and/or  $\bar{L}_2$ , say  $\bar{L}_0$  and  $\bar{L}_2$ , we shall re-evaluate the maximization process of (a),

(b) and (c) by replacing  $\bar{L}_1$  and  $\bar{L}_2$  with  $\bar{L}_1$  and  $\bar{L}_2$  on the right hand side of the equations in (1). The repetitive procedure as described here need not be a tedious one because a simpler procedure making use of parametric programming has already been worked out.<sup>5</sup> A similar parametric programming procedure for a change of relative prices of output is also available.

Graphically a simple expansion path can be illustrated. In figure 4a, we have an expansion of  $\bar{L}_1$  to  $\bar{L}_1$  on the assumption that there is no changes of  $\bar{L}_2$ . We extend OI to OI' and draw O' 'T' parallel to O'I. Before the resources expansion, intersection point c determines the extent of resources allocation between the two industries as demonstrated in Figure 3. It is now replaced by c' in Figure 4a. Similarly in Figure 4b we assume an expansion of  $\bar{L}_2$  to  $\bar{L}_2$ , but no increase in the resources of  $\bar{L}_1$ . The point of intersection c' of O' 'T' and OI replaces c as the point determining how the resources should be distributed between the two industries under the new assumed condition. The graphic illustration can be easily extended to cases where both resources may expand or the slope of OI or/and OT may change on account of new technology. There can be also a combination of such cases. But such extensions are straightforward and can be left to the interested reader.

Finally, balanced growth from the supply side, if properly interpreted, can and perhaps should always be a general equilibrium model. This is clear if one pushes the Marshallian formulation of the supply and demand framework to its logical conclusion.

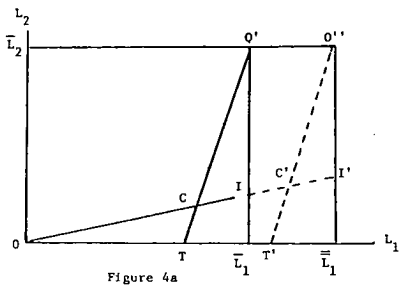


Figure 4a

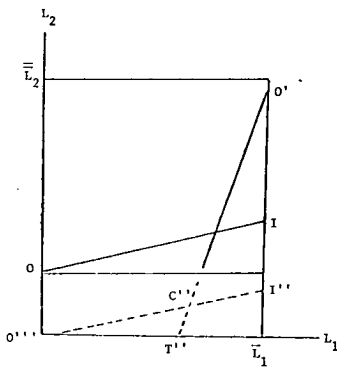


Figure 4b

NOTES

---

- The authors are professors of economics, Michigan State University.
  
- 1. For a survey of the concept see Jose Maria Dagnino-Pastore, "Balanced Growth: An Interpretation", Oxford Economic Papers, vol. 15 (July 1963), pp. 164-176. Discussions of Marshall's treatment of economic growth have not recognized his use of the idea; see, for example, Alexander John Youngson, "Marshall on Economic Growth", Scottish Journal of Political Economy, vol. 3, (February 1956), pp. 1-18.
  
- 2. Alfred Marshall, Principles of Economics, 8th edition, New York: Macmillan, 1920, pp. 272-273.
  
- 3. Ibid., p.489.
  
- 4. Under the assumed fixed proportion in factor combination, there is no guarantee of full employment of factors. This is because the iron-textile transformation frontier has a kink. Any point on the frontier will maximize the value of output for an appropriate vector of positive output prices. Similarly, any point can be tangent to the (convex) indifference curves of some social welfare function. But only at the kink are both factors fully employed.
  
- 5. S.I. Gass, Linear Programming, (New York: McGraw-Hill), 1958, ch. 8; and W. W. Garvin, Introduction to Linear Programming, (New York: McGraw-Hill) 1960, ch. 15.