

Keith Frearson On Roy Harrod, As Told To Geoff Harcourt

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Abstract: The first part reproduces the manuscript of Keith Frearson's hitherto unpublished notes on 'Harrod's dynamics', which constitute a prelude to his important article on this subject in the June-December 1964 issue of *Australian Economic Papers*. The second part is an Appendix in which Harcourt explains the significance for Harrod's system of the Oxford economist's abandonment of the autonomous consumption component in the Keynesian consumption function.

In the June-December 1964 issue of *Australian Economic Papers*, the late Keith Frearson published an article entitled 'Recent Developments in the Theory of Economic Growth'. It was much shorter and, superficially, more narrow in scope than Frank Hahn's and Robin Matthews's justly famous survey of growth theory in the *Economic Journal* of the same year. Nevertheless, the quality of Keith's analysis and the crystal clarity of his exposition made his article a worthy complement to the Hahn-Matthews survey. (In fact, Keith regarded his article as 'an examination of the problems involved in the formulation of a theory of economic growth'; it was 'not intended to be a survey of modern growth theories' (Frearson 1964, n. *, p. 1)). Keith singled out Arthur Lewis (1955) and Joan Robinson (1956) as authors of '[t]he two major books on the subject' (Frearson 1964, p. 1). He had taken 1950 as his starting point; otherwise, I am sure he would have included Roy Harrod's 1948 *Towards a Dynamic Economics* as well, for he was a great admirer of Harrod's contributions. Indeed, his discussion of the issues in his 1964 article centred around Harrod's contributions and the problems they raised. He cites Harrod's 1948 book.

When I was lecturing on growth theory in Cambridge in the early 1970s, Keith wrote out for me his understanding of Harrod in a set of notes with the title 'Harrod's Dynamics – the F.T.¹ Version (as told to G.C.H.)'

I still have his manuscript. It is such a clear and profound exposition that I thought it should be published as a fitting prelude to his 1964 article and in memory of Keith, who died on February 2, 2000.² I also include an appendix, which takes up a point raised by one referee of the present note.

Harrod's Dynamics – the F.T. Version (as told to G.C. H.)

1 Concerned with the rate of growth in total output, and so extends the Keynesian 'short-period' analysis into the long-run. Keynes was (formally) concerned only with the short-period "expenditure" effect of investment; Harrod is concerned as well with its long-run "capacity-creating" effect.

2 The r/g in Y is $g = \frac{\Delta Y}{Y}$ – so r/g in Y depends both on the *increase* in Y , and the *level* of Y from which that increase takes place.

3 So have two questions: what determines Y , and what determines ΔY ?

4 What determines Y ? This is the Keynesian analysis.

$$\text{Have } Y_A = C^0 + I$$

Where Y_A = actual level of output produced, C^0 = output of C-goods produced, and I = output of I-goods (all in value terms.)

So: (i) What determines C^0 ?

C^0 is determined by short-period expectations about the level of demand for C-goods in the current period. “But it will often be safe to omit express reference to *short-term* expectation ... expected and realised results run into and overlap one another in their influence ... and producers’ forecasts are more often gradually modified in the light of results than in anticipation of prospective changes.” (Keynes 1936, pages 50-51 [emphasis in original]). So for long-run analysis we can abstract from the short-period, Keynesian distinction between the output of C-goods and the demand for them, and assume that in each period $C^0 = C$. We can then write the level of output in each period as $Y = C+I$, where Y , the actual level of output, equals the “equilibrium” level of output in the Keynesian sense.

The simplest form of the Keynesian analysis can then be written:

$$Y = C+I$$

$$C = cY$$

I is autonomous,

$$\text{so that } Y = \frac{1}{1-c} I = \frac{1}{s} I.$$

This means that, *for given s*,

(i) The level of Y is determined by the level of I ,

(ii) The r/g in Y is determined by the r/g in I .

{The r/g in total output is determined by the r/g in the *demand* for output. If s varies, r/g in $Y = r/g$ in I less r/g in s .}

(ii) What determines I ?

Investment goods are produced in response to producers’ *long-term* expectations about the future level of demand – these expectations are determined by “animal spirits”. Investment constitutes an addition to the capital stock, and so increases the productive capacity of the economy. Clearly the increase in productive capacity generated by a given I corresponds to the *expected* increase in demand which induced that investment.

We can write $\Delta P = q_r I$ where ΔP is the increase in productive capacity due to I , and equals the *expected* ΔY which induced that I . q_r defines the relationship between the actual level of I undertaken in a given period, and the increase in productive capacity which it generates. (It is the reciprocal of Harrod’s v_r .) Obviously the higher is q_r the greater is the *potential* growth in output which a given period’s investment makes possible.

ΔP must be clearly distinguished from ΔY , the *actual* increase in output produced in response to an increase in demand. The relationship between the actual investment made, and the increase in actual output associated with it, is

given by $\Delta Y = q \cdot I$, where $q = \frac{\Delta Y}{I} = \frac{\Delta Y}{\Delta K}$ is the “observed marginal output-capital ratio”. q is the reciprocal of ICOR [the incremental capital-output ratio] – Harrod’s v .

Note: The foregoing could be expressed $\Delta Y = q \cdot I$ and $\Delta Y = q_r I_r$, where I_r is “required” investment – the investment which produces the increase in productive capacity “required” to sustain ΔY .

- 5 Investment in a given period is undertaken in anticipation of an increase in demand in the next period. But “it is of the nature of long-term expectations that they cannot be checked at short intervals in the light of realised results” so that “the factor of current long-term expectation cannot be even approximately determined or replaced by realised results” (Keynes 1936, page 51). Hence the problem arises that the expectations upon which investment is based may not be fulfilled, and the actual increase in output produced (in response to demand) may not correspond to the increase in productive capacity created.

If $\Delta Y < \Delta P$ (which means that $q < q_r$) then expectations are not fulfilled, and there is excess capital capacity.

If $\Delta Y > \Delta P$, ($q > q_r$) then expectations are overfulfilled (their cup floweth over), and there is a shortage of capital capacity.

Note that this means that q_r must be defined as a “normal capacity” coefficient. If this were not so, then we could not have $\Delta Y > \Delta P$, and ΔP would set a ceiling to the possible increase in Y .

Note also that inequality between ΔY and ΔP , since it expresses inequality between actual and expected increases in sales, is also a reflection of a difference between the expected rate of profit from a given investment (Keynes’s MEC) and the actual rate of profit obtained.

$\Delta Y > \Delta P$ means actual $r/p > \text{MEC}$

$\Delta Y < \Delta P$ means actual $r/p < \text{MEC}$

- 6 In this simplest of models, we have two basic equations.

1) $Y = C + I$, which leads to $Y = \frac{1}{s} I$;

2) $\Delta P = q_r I$

The actual r/g in output which takes place between any two periods

$\left(g = \frac{\Delta Y}{Y} \right)$ equals the r/g in I , and hence is an independent datum determined

primarily by entrepreneurial expectations (by “trial and error, by the collective trials and errors of vast numbers of people” – Harrod).

- 7 The observed (statistical) r/g in Y can be viewed from two standpoints.

(i) From the point of view of growth in the labour force *employed*.

$$\text{Since } Y = L \cdot \left(\frac{Y}{L} \right)$$

$g = l + r$
 where $l = r/g$ in labour force *employed*
 $r = r/g$ in labour productivity (O/man employed).

(ii) From the point of view of capital accumulation.

$$\Delta Y = q \cdot I; Y = \frac{1}{s} I; \text{ hence } g = \frac{\Delta Y}{Y} = s \cdot q.$$

Note: q is an *observed* value, which depends on g and s ; i.e., $q = \frac{g}{s}$.

8 Harrod at last?

We can then define two conceptual r/g s:

(i) The “warranted” r/g in output is the rate necessary to be achieved if the additional capital created by the investment is to be fully utilised, i.e., if $\Delta Y = \Delta P$. Then $q = q_r$, and the warranted r/g is defined by $g_w = s \cdot q_r$.

g_w is thus defined by the Propensity to Save, and the productivity of new capital. It is the r/g in Y necessary to be achieved if full employment of capital is to be maintained.

(ii) The “natural” r/g is the r/g necessary to be achieved if full employment of the labour force is to be maintained.

If $n = r/g$ in *available* labour force, then full employment requires that $n = l$, and the natural r/g is defined by $g_n = n + r$.

g_n sets an upper limit to the actual r/g of the economy in the long-run.

9 To create the *possibility* of growth with full employment of both labour and capital it is necessary that $g_w = g_n$, i.e., $s \cdot q_r = n + r$.

For this possibility to be *realised*, it is necessary that $g = g_w = g_n$.

From the growth point of view, the essential lecture [sic] of Harrod’s analysis is that it is concerned with the problem of maintaining over time the full employment of both labour and capital.

[G.C.H. – I have left out the “instability” problem of inequality between g and g_w – although it is easy enough to explain that if, for instance, $g > g_w$, then $q < q_r$, so $\Delta Y > \Delta P$, and there is a shortage of capital capacity – in which case Harrod *assumes* that the r/g in I will be increased, so increasing g . But you can make up all sorts of ‘adjustment mechanisms’.]

10 It is then possible to construct all sorts of interesting situations with inequality between g , g_w and g_n . One thing which I have found bothers all students (and some more illustrious people!) is: *why* does an increase in s increase g_w ? It is tempting to assume that it is because investment is increased, and so the r/g in productive capacity is increased. But not so! It is because increasing s lowers the *level* of Y associated with a given level of I .

11 The observed productivity of investment as reflected in q , the observed marginal output-capital ratio, is affected by:

(i) The degree of capacity utilisation, e.g., excess capital capacity lowers the value of q .

(ii) The level of technology. Improvements in technology will raise the value of q .

- (iii) The proportion of total investment devoted to Social Overhead Capital (“autonomous” or “unproductive” investment). The greater this proportion, the lower the value of q .
- (iv) The gestation period for investment. The longer is this period, the lower will be the value of q during that time.
- (v) The proportion of gross investment devoted to replacement. The higher this proportion, the lower the value of q .
- (vi) The price of investment goods in terms of consumption goods. (q is a *value* coefficient.) The higher this price, the lower the value of q . (Haig and Preston).³
- (vii) Structural shifts in the economy. If sectors with high sectoral q -values are expanding rapidly, the overall value for q will be increased. (cf Japanese Plan). This is what is discussed by Colin Clark (Growthmanship) [1961], Angus Maddison (Economic Growth in the West) [1964] and Solow (in Phelps [1968], The Goal of Economic Growth).

Apologies!

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Appendix

One referee commented on a draft of this note that, though Keith mentioned the issue, he never discussed how crucial it was for the derivation of Harrod’s result that Harrod dropped the autonomous consumption term from Keynes’s consumption function. I agree that it affects the simplicity of Harrod’s expression for g_w ; but does it significantly affect the deep insight that Harrod (and Marx before him) offered concerning the basic instability of the motion of unfettered capitalism? Let me elaborate. Following Sen’s exposition, we derive the expression for g_w as follows. Write the identities $S \equiv I \equiv \Delta K$. Assume that $S = sY$ *always*, where $s = mps$, i.e., the community is always on its saving function. Then we may write $\frac{sY}{\Delta Y} \equiv \frac{\Delta K}{\Delta Y}$, so that $\frac{\Delta Y}{Y} \equiv g \equiv s \frac{\Delta Y}{\Delta K} \equiv \frac{s}{q}$, where q is the incremental capital-output ratio and g is the rate of growth. Harrod’s investment function may be written as

$$I_t = q(X_t - Y_{t-1}) \quad (1)$$

where X_t = expected sales at time t , Y_t = realised sales and output at time $t-1$ and $(X_t - Y_{t-1})$ = the increase in expected demand that I_t is to cater for through q :

$$Y_t = \frac{1}{s} I_t = \frac{1}{s} q (X_t - Y_{t-1}) \quad (2)$$

where q is now the *desired* incremental capital-output ratio.

We assumed that actual income is always short-period equilibrium income, that is to say, we abstract from the groping process whereby any initial gap between planned investment and planned saving gives out stabilising signals which tend to take the economy towards the equilibrium point, Keynes’s point of effective demand:

$$\frac{Y_t}{X_t} = \frac{q}{s} \left(\frac{X_t - Y_{t-1}}{X_t} \right) = \frac{q}{s} \hat{g}_t, \quad (3)$$

where \hat{g} = expected rate of growth of sales.

If expectations were to be realised, the economy would be on g_w . What then is the condition for $\frac{Y_t}{X_t} = 1$ and the corresponding expression for g_w ?

$$\frac{Y_t}{X_t} = 1 \text{ if and only if } \hat{g}_t = \frac{s}{q}. \quad (4)$$

That is to say, $g_w = \frac{s}{q}$, Harrod's expression, and the expected, actual (g_t) and

warranted rates of growth all coincide. Moreover, $g_t \geq \hat{g}_t$ if $g_t \geq \frac{s}{q}$, i.e., actual growth exceeds, equals or falls short of expected growth if the latter exceeds, equals or falls short of warranted growth.

However, if there is an autonomous term in the consumption function, say A ($-A$ in the saving function, $S = -A + sY$),

$$Y_t = \frac{q}{s} (X_t - Y_{t-1}) + \frac{A}{s} \quad (5)$$

$$\frac{Y_t}{X_t} = \frac{q}{s} \left(\frac{X_t - Y_{t-1}}{X_t} \right) + \frac{A}{sX_t} \quad (6)$$

and when $\hat{g}_t = \frac{s}{q}$, $\frac{Y_t}{X_t} \neq 1$ because $\frac{A}{sX_t} \neq 0$. ($\frac{Y_t}{X_t} = 1$ when $\hat{g}_t = \frac{s}{q} - \frac{qA}{sX_t}$.)

In my view Harrod's most important insight may be put as follows. Having ruled out by assumption the stabilising properties of a gap between planned saving and planned investment in the short period, Harrod sensed the destabilising properties of such a gap for the long period. To illustrate this we present a simple

diagram which is essence of Harrod *à la Sen*. Our contention is that in the *Essay and Towards*, Harrod sensed the longer-period implications of discrepancies between planned saving and planned investment, implications that are obscured by considering only the employment-creating effects of the relationship between these two quantities. The implications relate to both the conditions for steady growth and the instability of the economy if these conditions are not attained.

The diagram shows clearly why the ordering referred to above,

$$g_t = \hat{g}_t \quad \text{if} \quad \hat{g}_t = \frac{s}{q} (= g_w), \text{ comes about and makes explicit the sense in}$$

which discrepancies between planned S and planned I are the basic cause of them. On the horizontal axis we measure Y_{t-1} , Y_t and X_t ; on the vertical axis, S and I . $OS(S_t = sY_t)$ is the saving function and $II(I_t = q(X_t - Y_{t-1}))$, the investment function. When $X_t = Y_{t-1}$, $I_t = 0$. The value of q is greater than the value of s , because it is not constrained to be less than unity and the periods of time that we are dealing with are such as to make q greater than unity.

As we mentioned above, actual income is always the short-period equilibrium level of income, i.e., it is the income associated with the level of saving that equals the level of planned investment, itself given by the II function in conjunction with given values of Y_{t-1} and X_t .

At X_e , Y_e , $I_t = q(X_e - Y_{t-1}) = sY_e (= sX_e)$ and

$$\frac{X_e - Y_{t-1}}{X_e} = \frac{Y_e - Y_{t-1}}{Y_e} = \frac{s}{q}$$

(expected = actual = warranted).

At X_1 , $X_1 < Y_1$

$$I_t = q(X_1 - Y_{t-1}) = sY_1 (\neq sX_1)$$

$$\frac{Y_1 - Y_{t-1}}{Y_1} > \frac{X_1 - Y_{t-1}}{X_1} > \frac{X_1 - Y_{t-1}}{Y_1} \left(= \frac{s}{q} \right)$$

(actual > expected > warranted).

At X_2 , $X_2 > Y_2$,

$$I_t = q(X_2 - Y_{t-1}) = sY_2 (\neq sX_2) \text{ and}$$

$$\frac{Y_2 - Y_{t-1}}{Y_2} < \frac{X_2 - Y_{t-1}}{X_2} < \frac{X_2 - Y_{t-1}}{Y_2} \left(= \frac{s}{q} \right)$$

(actual < expected < warranted).

Consider, first, the case when expected sales are equal to the value of income associated with the interception of OS and II , i.e., $X_e = Y_e$. Then investment expenditure is I_e and this produces an equilibrium level of income of Y_e , for at that level, $I_e = S_e$. In this case, expected sales and actual sales and income coincide. We then get:

$$I_t = q(X_e - Y_{t-1}) = sY_e (= sX_e),$$

which implies that the expected rate of growth equals the actual rate of growth which in turn equals the warranted rate of growth.

Now consider the case where expected sales are X_t . At X_t , planned $I >$ planned S , and so the equilibrium level of income, Y_t , is greater than X_t . That is to say, expectations of sales greater than those associated with the warranted rate of growth imply an actual level of income (and rate of growth) which exceed *both* the warranted and the expected level (and rate of growth) – short-period income must settle at that point if S is to equal I . We thus have:

$$I_t = q(X_t - Y_{t-1}) = sY_t (\neq sX_t)$$

$$\text{and } \frac{Y_t - Y_{t-1}}{Y_t} > \frac{X_t - Y_{t-1}}{X_t} > \frac{X_t - Y_{t-1}}{Y_t} = \frac{s}{q}.$$

$$\left(1 - \frac{Y_{t-1}}{Y_t} > 1 - \frac{Y_{t-1}}{X_t} > \frac{X_t}{Y_t} - \frac{Y_{t-1}}{Y_t} \right)$$

i.e., the actual rate of growth is greater than the expected because the expected rate of growth is greater than the warranted rate of growth.

Finally, if expected sales were less than X_e , we would get the reverse results: warranted $>$ expected $>$ actual.

All of these follow from the discrepancy between planned S and planned I at the *expected level of demand* X_t . Expected $I \oplus$ planned S implies actual $Y_t \oplus X_t$. It is these discrepancies which give rise to Harrod's discussion of stability. For Harrod, as a good Keynesian (without quotes), stresses the link between realisations and expectations. The realisation of a rate of growth of sales greater than the warranted and previously expected rates of growth could lead to an expectation of at least the last period's rate of growth of sales. This, in turn, leads to both the warranted and the expected rate of growth of sales being exceeded again, as a glance at our diagram will show – inflationary instability sets in. By similar reasoning, it is clear that if the expected rate of sales is the warranted rate (and is therefore achieved), and if this expectation is projected, steady growth at the

warranted rate will be maintained. If, finally, the achieved rate is *less* than the warranted rate, this will lead, on the same assumption about expectations, to deflationary (contractionary) instability. Moreover, though planned I is always realised, it is not what would have been planned had the businesspeople known the actual Y involved.

Now we draw the diagram again including a (negative) constant term, A , in the saving function; see Figure 2. A similar story of destabilising signals may be told; the economy will only be on and remain on g_w if expected sales are such as to give the values of $I_e (= S_e)$. I rest my case, as Keith would have said.

Notes

1 F.T.: Frearson Trained.

2 I wrote a memoir of Keith for the *Economic Record* (Harcourt 2000), in which I mentioned the existence of the notes. As they illustrated so well Keith's extraordinary powers as a teacher (and kindness as a friend), I thought it would be good to have them in the public domain. The notes were a great help to me when I prepared my lecture on Harrod. One referee asked what I thought was 'the best simplified (?) general Intro to Harrod'. I referred the students (second year ('Prelims')) to Amartya Sen's 'Introduction' to his selected growth *Readings* (1970). For the lecture I translated Sen's algebra into a saving-investment diagram and drew on Keith's insights in my exposition.

3 Despite wide-ranging searches and enquiries, I have not been able to track down this reference. GCH.

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