Masters of Theory and its Relevance to the History of Economic Thought[1]

Warwick, A. *Masters of Theory. Cambridge and the Rise of Mathematical Physics*. Chicago and London: University of Chicago Press. 2003. ISBN 0 226 87374 9 (cloth); 0 226 87375 7 (pb). US$95.00 (cloth); US$32.00 (pb).

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1 Introduction

One of the most important texts to be published in 2003 that is of relevance to the sub-discipline of the history of economic thought was written not by an historian of that sub-discipline, but by an historian of mathematical science. Andrew Warwick's *Masters of Theory: Cambridge and the Rise of Mathematical Physics* has nonetheless been largely overlooked by historians of economic thought, and, for this reason, a lengthy critical review of this work is warranted. Warwick has written what many mathematical purists would incorrectly dismiss as a 'contradiction in terms', namely, a cultural history of mathematics. He demonstrates that the cultural processes that governed the way in which Cantabrigian undergraduates sitting for the Mathematical Tripos initially came to grips with their subject matter had some bearing on the mathematical product that eventually appeared on the published page. The mathematical theorems, lemmas, proofs and applications were, in other words, not spun out of the aether by unadulterated minds sitting in clean Euclidean space, but were indirectly the product of the way in which the raw undergraduates were originally transformed into practising mathematicians, or, more usually, journeyman scholars or professionals with a mathematical bent. Such a cultural interpretation of an intellectual product is now pretty much standard in the history of physics, biology and economics, but it still provides a little *frisson* when applied to the field of mathematics, which is normally fêted as the purest of all the sciences. This intellectual thrill is further spurred by Warwick's patent rejection of the usual amateurish route taken in these sorts of studies. Specifically, instead of making a few lame references to Thomas Kuhn's tired concept of a paradigm before delivering some sweeping statements that are supported by no more than one or two protocol statements drawn from secondary references, Warwick has undertaken a breathtaking amount of original research to reconstruct the actual pedagogical machinery used to train students at Cambridge. He, of course, draws upon Kuhn (how could he not?), but he quickly moves on from the well-worn references to 'incommensurability' and 'tacit knowledge' to the far more difficult historiographical exercise of making his case.

The end result is a near seamless narrative of the history of Cantabrigian mathematical training that is accessible, with a little effort, to those from the wider history community who are not mathematically inclined, or at least not adept at the idiosyncratic style of mathematics that was deployed in the Victorian period. (Try reading James Clerk Maxwell's, 1873, *Electricity and Magnetism!*). This accessibility is particularly important for historians of economic thought, since most of the Cambridge economists from this period were initially trained in the

Mathematical Tripos (from Whewell to Fawcett to Marshall to Keynes) and yet, presumably because of the arcane nature of this Tripos, there is still no *systematic* study of how this early training guided their speculations within the field of economics. There are, of course, scholarly accounts of the mathematical training of specific Cambridge economists, such as in Peter Groenewegen's *A Soaring Eagle. Alfred Marshall 1842-1924* (1995, chapter 4), and good, but brief, descriptions of the Mathematical Tripos in larger works on the history of mathematical economics, as presented in Roy Weintraub's *A History of Mathematical Economics* (2002, chapter 1). But these and similar histories do not consider the influence of the early mathematical training on each and every Cambridge economist (see pp. 84-5 below for a comprehensive list of the Cambridge economists who completed the Mathematical Tripos). The authors of these works obviously have different (and equally worthy) objectives, and hence fail to trace the extent to which the mathematical and economic cultures of Cambridge overlapped. It is for this reason that the main objective in the critical review that follows is to show the way in which Warwick's findings may be relevant to such a study. The first step in this undertaking is to delineate the Mathematical Tripos itself.

2 The Mathematical Tripos

The English faith in competitive examinations was rendered concrete during the nineteenth century and it remained sufficiently strong over the years to survive, in a battered sort of way, the repeated attacks on the meritocratic principle during the post-modern malaise of the late twentieth century. This trust in the examination process from the age of Palmerston to the age of Blair was perhaps matched only by the confidence of the Imperial Chinese in the tests they employed to select the officials of the nine grades. The most trying and prestigious of all the examinations in this golden age of meritocratic idealism in England was the Mathematical Tripos examination held in Victorian Cambridge. This annual Cambridge climacteric became the model for the examinations introduced in the wake of the Civil Service reforms of 1871, which, in turn, became the template for all those exams throughout the Empire that were specifically designed to construct an élite based on intellect and character rather than birth and connection (Snow 1966, p. 32; Gascoigne 1984). The precise structure of the Mathematical Tripos, as it was practised in this era, is reconstructed at length in the book under review and, given that readers may already be acquainted with some of its basic features via the brief accounts in the aforementioned works by Groenewegen (1995) and Weintraub (2002), only those elements that are germane to my immediate objectives need be described in this critical review.

The Mathematical Tripos evolved throughout the nineteenth century, but its key features may nonetheless still be brought into sharp relief by taking the 1880s, which was the high tide of the degree, as a reference point. By this time it was the title given to a ten-term degree offered at the University of Cambridge that concluded with a gruelling examination constituted by 16 papers taken on nine days spread out over a two-and-a-half-week period. The first six papers were entitled Mathematical Tripos Part I and, being of an elementary nature, were designed as a filter mechanism to determine which students could proceed to the rest of the more complex papers, which were entitled Mathematical Tripos Part II. The students were placed in strict order of merit, where merit was determined objectively by the total marks scored in the examination. Those whom we would now refer to as the first-class honours students were (and indeed still are) called wranglers, the second- and third-class students were referred to as senior and junior optimes respectively, and the non-reading men who did not pursue these honours were known as the *hoi polloi* (which is Greek for the many) and it is from this term that 'poll-men' was derived (Warwick 2003, p. 55). All of these classes of graduates were literally awarded a Bachelor of the Stool, since it is believed that the term 'Tripos' derives from fact that students were, in the distant past, placed on a three-legged stool during a mock disputation on graduation day (Latham 1879, p. 778; Ball 1918, p. 312; cf Weintraub 2002, p. 12). Shortly after the awarding of this degree – which students first had to take if they wished to enrol in Classics (1822) and which long remained the dominant degree after the rise of the so-called 'Minor Triposes' of Moral Science (1848) and Natural Science (1848) – there was a second examination for the Smith Prize. This was less understood by the non-Cambridge community, and hence less valued, but it was generally accepted by mathematicians as a more striking indication of a student's mathematical capability since both the knowledge acquired via classroom drilling and the pace in the examination room counted for less (Gooch 1920, p. 29).

The first characteristic relating to this Mathematical Tripos that readily emerges from Warwick's study is that the very title of the degree was something of a misnomer during the Victorian period. The syllabus that then constituted this degree was effectively mathematical physics, and hence applied mathematics, rather than pure mathematics in the modern sense (the disciplinary scope of which is, by the by, more than usually difficult to define and delimit). The candidates were forced to master problems set in the physical world that were of considerable difficulty, but the majority of which could be varied in only a certain number of ways and hence which could be successfully solved by a very bright individual who had the stamina to devote many hours to preparatory bookwork and coaching sessions. It was, in other words, more a test of a candidate's work ethic, mental doggedness and ability to perform intellectual gymnastics under duress than the calibre or originality of his mathematical mind. Indeed, it was then generally accepted that the Mathematical Tripos was not a training ground for mathematicians at all, but a liberal education that prepared men for either life in general (if one held the backward-looking anti-utilitarian outlook) or a middle-class profession (if one embraced the 'vulgar' utilitarian vision then on the up). It was believed that the Tripos man was tailor-made for a sinecure in the form of a fellowship at one of the colleges, a stall in the editorial stables of the select Higher Journalists, underemployment at the bar, or the limited number of administrative posts throughout the Empire. If a senior wrangler subsequently transformed himself into a first-rate mathematician, like G.H. Hardy (fourth wrangler in 1898), or became adept at applying mathematics to make path-breaking contributions to physics, such as James Clerk Maxwell (second wrangler in 1854), or indirectly drew upon mathematics to revolutionise political economy, as did Alfred Marshall (second wrangler in 1865), then that would be a positive spin-off, not the central objective.

The 'liberal arts' interpretation of the Mathematical Tripos was founded on the reasonable pedagogical contention that the Cambridge man who had the logical faculties to navigate a Tripos question was also capable of solving the problems that would arise in any of the endeavours he wished to pursue. The wide acceptance of this philosophy also partly explains why many from the Cambridge hierarchy so vehemently defended the applied nature of the syllabus against those who wished to introduce more abstract mathematical components. They argued that the candidates most readily learnt how to reason from premises to conclusions when they believed that the premises were somehow self-evidently real, such as force in Newtonian mechanics or space in Euclidean geometry, and when the solving of the logical problems seemed to yield some insight of practical value. After all, if this was not the case, the students could just as easily have improved their problem-solving abilities by studying pure logic. It was further argued, and this contention has a familiar ring to an economist's ears, that the candidates could more readily intuit the nature of the problems if the calculations were preformed at least partly via geometric constructions rather than the symbolic manipulations used on the Continent (Warwick 2003, pp. 101ff). These conceptions of the Mathematical Tripos, which were perhaps most vocally defended by William Whewell, were increasingly attacked (and indeed successfully eroded) by a reform party as the century progressed, but they were not effectively overthrown until the Edwardian era and even then the degree remained applied in nature if judged by modern standards. Students who sat the Tripos in the 1950s still learnt just as much physics as pure mathematics and, worse, had obstacles placed before them when they strove to specialise in one or the other.

The second defining characteristic of the Mathematical Tripos that becomes apparent on reading Warwick's study is the importance of the coach. Although the college tutors provided elementary lessons in mathematics for the students under their charge, the pace and complexity of the delivery of the material was invariably set by the least able student. The candidates in pursuit of honours were therefore sent off to private mathematical tutors, known as coaches or 'pupil mongers', who provided intensive training for a fee. These coaches, who were themselves senior wranglers from previous years, treated their 'team' of candidates as thoroughbred race horses, knew the state of the track and the form of the runners, and had knowledge of the strange obstacles imposed by the stewards on any given race day. The very term coach was derived from the much admired 'coachman', who, with great skill and athletic prowess, oversaw the team of horses that drew the fast coaches that ran from London to Cambridge along the recently macadamised roads (Warwick 2003, pp. 90ff). Coach, along with team, thereafter quickly entered the national vernacular, appeared in Victorian novels such as Thackeray's *Pendennis* (in which Oxbridge and Camford were also coined) and was carried over to the sporting arena, where similar but less intense competitions were played out. As one of the earliest historians of the Mathematical Tripos has pointed out, the scandal of the system consisted in the fact that a candidate was 'compelled to pay a heavy fee to the University and his College for instruction, and yet found it advantageous at his own expense to go elsewhere to get it' (Ball 1918, p. 310). The advantage of the system, on the other hand, consisted in the fact that a candidate received the best training in mathematical physics in Victorian England that ready cash could buy.

Warwick places particular emphasis on the way in which some of the leading coaches, who were known as 'wrangler makers', employed distinctive teaching devices to construct an oral tradition within Cambridge mathematics. The candidates did not learn problem-solving techniques merely from textbooks and private study, but by watching a brilliant coach 'talk and chalk' his way through problems on a blackboard and 'reveal' how mathematical tools could be applied to a range of different problems from seemingly different physical domains (Warwick 2003, p. 228). Success in the Tripos was, indeed, rarely possible without a coach, and eventually success was even deemed unlikely if a candidate did not seek out a famous coach. The original ‘wrangler makers’ were William Hopkins and John Harmers, who in turn produced the second generation of coaches, such as Isaac Todhunter, William Steele and Edward Routh, who then produced the third generation of coaches, such as R.R. Webb. This pure ‘blood line’ perhaps peaked in pedagogical terms with Routh, who was easily the most famous of all of these coaches. Seven of the ten senior wranglers were members of Routh’s team in 1862, and approximately 80% of the top three wranglers were his men between 1865 and 1888 (Warwick 2003, p. 233). His name is now connected with the standardisation of teaching mathematics not only at Cambridge, but in the entire Empire, since his students imitated his teaching techniques (and the associated tacit knowledge conveyed in his classes) as professors and schoolmasters in Cape Town, Toronto and Sydney. The age of Routh and his fellow wrangler makers began to fade (and then only slowly) from the 1880s onwards, when a system of inter-collegiate mathematical lectures was introduced.

A third important characteristic of the Mathematical Tripos that is emphasised by Warwick is the remarkable strain, spectacle and status associated with the degree. Candidates invested huge amounts of physical and psychological effort to gain a place in the highly competitive scramble. Candidates outlaid this emotional capital partly because the rewards for a high wrangler position, in the form of a fellowship and reputation in the market-place, were immense. Leslie Stephen estimated that an individual could win £5000 who was so placed in this Tripos (1865, p. 38; see Moore 2002). Candidates also invested their all in the Tripos examinations because of the status earned from winning a senior position in this competition. It was seen as the greatest intellectual achievement in the land, with the nearest modern equivalent in terms of kudos being perhaps a Nobel Prize (or, given the stamina required, an Olympic medal). The associated customs and rituals were suitably full of pathos, theatre and the bizarre. The announcement of the order of merit on the Friday following the completion of the exams was an elaborate ceremony and reported in the national press as one of the more important newsworthy events in the Empire. The list was (at least from the 1860s) solemnly read out from the east balcony of the Senate House and then thrown down to the eager audience, who fought for a copy (see Warwick 2003, chapter 4.5). On the Saturday the degrees themselves were distributed in order of merit and the last of the junior optimes had a large wooden spoon lowered over his head from ropes strung from the balconies. He then good-naturedly paraded it through the streets of Cambridge (Warwick 2003, p. 57). It has been speculated that the spoon represented a malting shovel, and hence the student’s preference for beer over studies. It is, of course, also the source of the sporting term ‘winning the wooden spoon’. The more staid of the senior academics naturally did not take kindly to this celebration of mediocrity and, in 1882, a running mêlée took place in an attempt to prevent the students smuggling in the spoon for their informal ceremony (Warwick 2003, p. 211). The senior wrangler was, by contrast, honoured in a more formal and sober manner. Torch-lit parades were held in his home town; postcards were printed of the more famous candidates (!); and *The Times* continued to publish his biography, as well as the strict order of merit, until the order of merit was replaced by alphabetical listings in 1909.

The hours of bookwork each candidate needed to invest to remain competitive in the pursuit of these honours, together with the constant stress generated by the intense competition, naturally induced Cambridge undergraduates to take up physical exercise. These athletic pursuits, which are described in great detail by Warwick, both released the nervous tension that arose from the hours of study and developed the physical body to the pitch required for the candidate to sustain the intellectual marathon. The mania for rowing and walking was particularly intense, especially by the 1860s. Some famous walks, such as the 'Grantchester Grind' and 'Wrangler's Walk', reflect their origins as popular routes taken by reading men for their daily constitutionals, while inter-collegiate rowing competitions grew to the point where they were described in detail in the national press (Warwick 2003, p. 192). Indeed, these athletic endeavours quickly developed into competitive events that mirrored the intense competition of the Tripos itself. The tutors and coaches even advised their students to adopt these physical activities to improve their chances of academic success and, since nearly all were themselves graduates of the Tripos, carried on their undergraduate athletic pursuits (as well as more expensive ones, such as mountaineering) throughout their academic careers. The culture of scholarship and athleticism subsequently became intimately intertwined within the Mathematical Tripos and, given the way this Tripos dominated the university, in the wider Cambridge culture. It also should be emphasised that the emergence of this athletic culture was accompanied by the development of a manly temper amongst the ranks of the student body. It was believed that success in the Tripos was just as dependent upon a 'man' exhibiting character in the face of adversity as it was upon intellect, and candidates and spectators alike had little time for those who used sentimental notions to question a game that mirrored life, in which only the fittest survived. In short, Warwick's research on this subject provides a new perspective on the Victorian adoration of the muscular Christian (or more usually, the muscular Agnostic). This predilection has previously been attributed to the reforms within the public schools or to the demented preferences of particular social circles (such as the Fawcett-Stephen set) at Cambridge (see Collini 1989, 1991; Moore 2002). It is now clear, however, that the athletic and manly culture was far more widespread in Cambridge itself, and the importance of the Mathematical Tripos as a Victorian institution induced the export of these attitudes to the rest of the Empire.

The fourth characteristic that defined the Mathematical Tripos was the way in which the cultural practices that were specific to Cambridge shaped the very nature of the mathematical research produced there. Warwick's account of the way in which the cultural shifts within Cambridge changed the nature of the mathematics through time is convincing – although I must admit that the rhetorical force of this argument is dissipated somewhat by his tendency to return randomly to the point, in an explicit sense, over the 572-page text. This failing is, for the most part, the natural consequence of devoting many pages to the worthy task of building the concrete environment in which the changes took place, and hence it is a failing worth bearing. The most obvious and easily comprehended pedagogical shift that changed the nature of Cambridge mathematics was the move that took place some time in the mid-eighteenth century from an examination process based on oral communication and disputation to one based on written answers to problem sets (Warwick 2003, pp. 116ff). One can easily imagine the limited complexity of a syllabus that was designed to prepare a student for an oral exam, as it was in the Senate House in the 1750s, and the scope for increasing the complexity of the subject matter once students undertook paper-based training for a paper-based exam. Students ceased to defend general propositions on Newtonian laws, such as the extent of their truth, and began to employ long and technical chains of mathematical reasoning to solve problems relating to the physical world. One can also easily imagine the change in the skill set that students needed to thrive in the new teaching-cum-examination environment. Success ceased to be dependent on oratory skill, physical bearing, genteelness and a disposition for pleasant private reading and, instead, became dependent on an individual's competitiveness, mental agility, athletic endurance and capacity for bookwork.

The implications of the other main pedagogical shifts are a little more difficult to discern amongst the rich tapestry of the historical particulars presented by Warwick, but they are nonetheless persuasive when excavated. All of the shifts seem to relate to the rapid inflation in the technical difficulty of the Mathematics Tripos during the nineteenth century, which, in turn, was fuelled by the cultural developments described in the previous paragraphs, such as the introduction of the coaching system, the increased stamina induced by the rise of athletic pursuits, and the heightened competitive spirit in which the students chased the ever more valuable prize of a wrangler title. The list of possible connections between this escalation in technical sophistication and the actual research is lengthy, so only a selection of those connections that I found particularly convincing is presented here.

First, the more advanced questions within the Tripos examinations became so 'state of the art', and the Tripos examinations themselves became the centre of Cambridge life to such an extent, that senior mathematicians, including the great Lord Rayleigh, began to announce their mathematical research as specific problems in the annual examination. Perhaps the famous illustration of the issue of a new idea in this manner was Stokes's Theorem, which was set by George Stokes in the Smith Prize examination in 1854.

Second, to the consternation of a number of mathematicians, the actual research papers published by Cambridge mathematicians began to take the form of model examination problems. This development is not surprising given that all of their training involved solving problems in preparation for a problem-based exam and given that they were often set on a line of inquiry after tackling a novel exam question (Warwick 2003, pp. 158-9).

Third, if the mathematical research was not initially presented as an exam question or resembled an exam question when published, the Tripos examination (together with the coaching system) was certainly the means by which key research advances were interpreted, propagated and elaborated upon. The three most important mathematical physics books from the late 1860s and 1870s – William Thomson's and Peter Guthrie Tait's *Natural Philosophy* (1867), James Clerk Maxwell's *Electricity and Magnetism* (1873) and Lord Rayleigh's *Theory of Sound* (1877-8) – were all integrated within the mathematical research programme via the coaching and examination process (Warwick 2003, p. 324).

Fourth, mathematicians gained prestige as writers of textbooks with problem sets attached. This certainly contributed to the fame of Isaac Todhunter and Joseph Wolstenholme. The latter, who was a life-long friend of Leslie Stephen's and the model for the opium-smoking Mr Woolly in Virginia Woolf's *To The Lighthouse*, has been absent-mindedly dismissed within the secondary literature as a textbook writer, but it is now clear that this vocation was greatly respected at the time and that Wolstenholme's fame was derived from research published in the form of exam and textbook problems.

Finally, the coaching system of preparing students to deploy the same mathematical tools to answer problems from a number of different physical domains, encouraged research graduates to deploy (almost immediately) mathematical tools developed for one part of mathematical physics in another branch (Warwick 2003, p. 321ff). The techniques developed in Rayleigh's *Theory of Sound* (1877-8) were, for example, soon employed to comprehend any vibrating system.

It is clear, then, that Warwick has not written a traditional history of ideas in which the succession of theories within a discipline is faithfully traced. He has provided a history of mathematical training that explains how some individuals who were inducted at a particular site were transformed into knowing insiders while other individuals who were educated at other sites were made ignorant outsiders, and, further, he has delineated the shifts in cultural forces that changed this training process and hence changed the nature of mathematical research pursued by the knowing insiders at a particular site over time. Warwick also demonstrates that the accepted ways of doing and viewing mathematics were, in Kuhnian fashion, absorbed tacitly by protracted routines of learning that were embedded in the physical and cultural landscape of Cambridge. The end result was that mathematical papers were written in such a way that they could only be read with ease by other insiders. A mathematical paper or problem that would take a recent Cambridge graduate a few hours to master would take others – whether they be laypersons or mathematicians trained at other sites – days, weeks or a life-time to comprehend. This near incommensurability became a more serious problem as Cambridge mathematics escalated in sophistication as the years progressed. Augustus De Morgan (fourth wrangler in 1826), who trained William Stanley Jevons in mathematics, complained from his base at University College in 1855 that the texts written by Cambridge mathematicians were becoming incomprehensible to non-Cambridge students (Warwick 2003, p. 152). Indeed, the insider-outsider dichotomy, which Warwick refers to as the Great Divide (Warwick 2003, p. 9), was overcome in an effective sense only when Cambridge graduates were, along with their tacit knowledge and cultural practices, exported to the public schools and universities throughout the Empire. The task now before us is to determine the extent to which both this tacit knowledge and the cultural practices were exported to the discipline of economics when a succession of men trained in the Mathematical Tripos migrated to this developing discipline.

3 The Implications For Economics: The Players

The role played by the Mathematical Tripos in the development of economic theory, and in particular for the evolution of economics at Cambridge, should be self-evident from the sheer number of the Cambridge economists (and the Cambridge scholars who dabbled in economics) who completed this degree. George Pryme was sixth wrangler in 1803. William Whewell was second wrangler and second in the Smith Prize in 1816. Henry Dunning Macleod was a senior optime in 1843. Leslie Stephen was twentieth wrangler in 1854 (cf. Groenewegen 1995 and Whitaker 1996). Leonard Courtney was second wrangler in 1855 and joint winner of the Smith Prize for that year. Henry Fawcett was seventh wrangler in 1856. Charles Baron Clarke was third wrangler in 1856. Henry Sidgwick read for both the Classical and Mathematical Tripos and became thirty-third wrangler in 1859. Alfred Marshall was second wrangler (to Lord Rayleigh) in 1865. John Neville Keynes was initially enrolled in the Mathematical Tripos, but, suffering from painful toothaches and depression, transferred to the Moral Science Tripos and became senior moralist in 1875. William Earnest Johnson was eleventh wrangler in 1882. Arthur Berry was senior wrangler in 1885. Alfred William Flux was bracketed senior wrangler in 1887. Arthur Lyon Bowley was bracketed tenth wrangler in 1891. Charles Percy Sanger was second wrangler in 1893 and gained a first in the Moral Science Tripos in 1894. Ralph George Hawtrey took on too many extra-curricular activities and was eighteenth wrangler in 1901. John Maynard Keynes also spread himself too thinly and earned (what was for him) the disappointing rank of twelfth wrangler in 1905.

It seems that the Cambridge economists who did not have a background in mathematical physics were the exception that proved the rule. Individuals such as Herbert Somerville Foxwell (senior moralist in 1870), William Cunningham (senior moralist in 1872) and John Shield Nicholson (a first in the Moral Science Tripos in 1877) are marked out by their lack of knowledge concerning mathematics and, in particular, by their ignorance of the idiosyncratic mathematical physics pursued at Cambridge in the age of Victoria. It also must be emphasised that even as it became increasingly acceptable to take one of the minor Triposes towards the end of the nineteenth century, the best undergraduates were still drawn to the Mathematical Tripos and, more importantly, the most promising Cambridge economists still took up economics via this route. Marshall, for example, famously lamented that the best students of economics were not coming to the discipline via the Moral Science Tripos (in which economics then resided at Cambridge). The prospective economists were, instead, Mathematical Tripos students who either audited his classes or simply talked to him about the content of his *Principles* (either in its draft or published form). In Marshall's words:

> Were it not for men as Berry Flux & Bowley who do not take the Tripos at all, & who learn what they do from me chiefly in private conversation, it would be little better than hack work to teach Pol Econ here. Sanger is the only student (man or woman) who has taken up economics for Part II & was really worth teaching. But one Sanger, or even one Bowley, is a good recompense for 5 years work; & I am content. (Marshall to J.N. Keynes, 2 November 1895, in Whitaker 1996, volume 2, p. 133)

It also should be noted that most of the winners of the economics essay prizes in the 1880s and 1890s were from the Mathematical Tripos (not the Moral Science Tripos) and that it was the superior Mathematical Tripos men who rose above the grind of their day jobs to actually make a contribution to the economic literature after their graduation. Marshall singled out Bowley on this last score, observing that Bowley had worked very hard as schoolmaster after taking his degree in 1891, yet had still made a contribution to the literature by 1895 that was sufficiently important to earn him a medal from the Statistical Society (Marshall to E. Sidgwick, 16 February 1896, in Whitaker 1996, volume 2, p. 159).

The Cambridge economists themselves were certainly cognisant of the fact that they were part of a traditional migration between the two disciplines. There was an unbroken thread linking each successive migrant. Fawcett famously challenged Whewell on the issue of the suitability of the inductive method in economics at the British Association meeting of 1860. Fawcett, in turn, was Keynes senior's avuncular confidant and used all of his rhetorical powers to encourage him to complete the Mathematical Tripos in 1873 (see Deane 2001; Moore 2003). Keynes subsequently became Marshall's lieutenant in the professionalisation of economics at Cambridge in the 1880s and 1890s. Marshall (who was more aware of intellectual lineages than anyone) then developed the talents of Flux, Bowley and Sanger in the 1890s. Many other linkages could be delineated, but that would be the task of a larger (still unwritten) work on the oral tradition in the economics of Victorian Cambridge. It is enough to state here that the culture (in the broadest meaning of that term) of the mathematical physicists at Cambridge left its indelible mark on the culture of economics at Victorian Cambridge. Indeed, it was only in the late 1890s and early 1900s that men without a mathematical physics background started to dominate the ranks of the juniors entering the economics profession. The new recruits were literary-minded men, such as Arthur Cecil Pigou (Historical Tripos in 1899), John Harold Clapham (Historical Tripos in 1895) and Sydney John Chapman (Moral Science Tripos in 1897-8). It was also around this time that the Mathematics Tripos began to lose its hold on Cambridge itself. More Triposes (including, of course, economics) were introduced and the Historical Tripos eventually became the degree of choice. For every Frank Ramsey, there were now several individuals like Roy Harrod and Nicholas Kaldor who struggled with mathematics and hailed from diverse backgrounds with regard to training. The thread that ran through the successive generations of mathematical physicists who dominated Cambridge economics was broken.

4 The Implications for Economics: The Two Main Generations

The list of players presented in the last section, together with their year of graduation, indicates that there were two obvious injections of mathematical talent (in terms of numbers) into the discipline of economics. Specifically, there was the generation of the 1850s and the generation of the late 1880s and early 1890s. Neither of these two generations was equal in importance to the great ‘one-off’ exports from mathematical physics, such as a Marshall or a Keynes, but they are still of extreme interest to the historian as they indicate that there was a community of scholars with a common background and language in mathematics entering the domain of economics at the same time. Consider each generation in turn. The 1850s generation, which has already been made a subject of study (see Moore 2002), should be interpreted as a circle of close friends who were preoccupied with the social and economic problems raised by John Stuart Mill in his holy trilogy of ***Principles***, ***Logic*** and ***Liberty***. This circle, which will hereafter be referred to as the Cambridge Millites, was initially constituted by an undergraduate set that included Fawcett (1833-1884), Stephen (1832-1904), and Clarke (1832-1906). Courtney (1832-1918) subsequently became thick with Fawcett and Stephen in London in the 1860s and the 1870s, while Sidgwick (1838-1900), who was slightly younger and more of a satellite to this social set, became acquainted with all of these individuals through the societies and clubs to which they all belonged over the same period. These individuals published substantial (if sometimes uninspiring) works on the theory and history of economics; took up important institutional positions within the discipline, such as Professor of Political Economy at University College London (Courtney) and Professor of Political Economy at Cambridge (Fawcett); figured prominently in the clubs in which economics was discussed, such as the Political Economy Club and the Radical Club; and interacted with the senior economists of the previous generation, such as John Elliot Cairnes, William Thornton and the great Mill himself. Their published works were, however, paradoxically defined by the traditional narrative form and rhetorical devices of the

Millite age. They were strangely bereft of any explicit use of mathematical symbols or the wholesale lifting of models from mathematical physics.

The second generation, by contrast, had no qualms about deploying in the field of economics the mathematical methods and tools on which they had been weaned within the Mathematical Tripos. This cohort, which has yet to be analysed as a single entity within the literature, was not a social set bound tight by their common social experiences at a particular Cambridge college. They graduated in different years, resided in different colleges and belonged to different social sets. They were, however, all introduced to economics via Marshall's *Principles* and knew each other intimately. The main figures included Berry (1862-1929), Flux (1867-1942), Bowley (1869-1959) and Sanger (1871-1930). Each of these young men came under Marshall's tutelage, and, indeed, all were asked to appraise the drafts of the first or successive editions of Marshall's *Principles*. They all subsequently made a contribution to mathematical economics and statistics. Berry became a respected mathematician at Cambridge and taught a course entitled 'Diagrammatic and Mathematical Treatment of Economic Theory' from 1891 to 1900. Flux almost absent-mindedly introduced economists to Euler's Theorem in a review of Wicksteed's *Essay on the Co-ordination of the Laws of Distribution* (1894). He also wrote the first Marshallian-driven textbook (1904, which had mathematical appendices) and eventually emerged as a leading applied statistician, making contributions to the literature devoted to index numbers and national income. Bowley, who won the Cobden Prize in 1892 and the Adam Smith Prize in 1894, wrote one of the first mathematical economic texts (*The Mathematical Groundwork of Economics*, 1924) and dominated statistical research in England for the first half of the twentieth century. Sanger, however, is perhaps the most interesting and underestimated of this generation. His survey of Italian contributions to mathematical economics in the *Economic Journal* in 1895 was for many years seen as the high-water mark of technical sophistication in economics (Whitaker 1987, p. 242); he often examined for the Economic Tripos; he taught part-time at University College and the LSE while practising as a successful Chancery barrister; and he wrote over fifty reviews for the *Economic Journal* between the 1890s and 1920s. John Maynard Keynes compared him to the great Frank Ramsey: 'both mathematicians by metier who were strongly drawn to economic science' (Keynes 1930, p. 154).[2]

The historiographical conundrum (which cannot be examined at any length in this short space) is why the later generation so readily embraced mathematical economics, while the earlier generation baulked at deploying what they had learned as undergraduates. This sort of analysis must, of course, be preceded by the admission that the use of mathematical thinking need not take the form of implicit functions, derivatives and proofs. The diagrammatic representation of supply and demand in Cartesian space is just as much mathematics as the implicit functions they represent, while the verbal rendering of functional relationships in economics is simply mathematical economics once removed. I am with Jevons on this score: 'economists have long been mathematicians without being aware of it' (Jevons 1879 [1931], p. xxiii). It also must be emphasised that the mathematical physics training can easily be excavated from the written work of the earlier generation. It is known, for example, that Stephen constructed supply and demand diagrams (now lost) in the early 1860s and that he tore strips off Macleod (without actually advancing many substantive arguments) when the latter deployed sloppy mathematics in his economic writings (see White 2004). Still, in spite of this, there is no doubt that the earlier generation of Cambridge economists did not embrace mathematical economics in the ‘full-blown’ sense that the later generation did. The most interesting hypothesis that explains this historical outcome derives from Warwick’s research, namely, that many of those who enrolled in the Mathematical Tripos did so ***not*** because they possessed mathematical dispositions, but because it offered the greatest intellectual prize in the land for those who could master the curriculum through bookwork. Those who subsequently excelled in this endeavour, though clearly brilliant individuals, were not necessarily mathematicians made in heaven. It could be, then, that the earlier generation succeeded in the Mathematical Tripos through hard work, while the later generation succeeded via both hard work and an innate mathematical vision. Stephen was certainly surprised by his own success in the Tripos examinations, Fawcett treated the entire process as a game through which he could demonstrate his logical faculties, while Clarke’s true vocation was botany. Flux and Bowley were, by contrast, wranglers with obvious mathematical talent, while Bertrand Russell attested to Sanger’s mathematical brilliance by noting that the latter (who was Russell’s study partner) had usually completed a problem before he had understood the question (Russell 1967, p. 57). However, before leaving this tentative (and near tautological) hypothesis concerning what separated the two generations – namely, that ‘the early Cambridge scholars failed to embrace mathematical economics because they were poor mathematicians’ – it also must be admitted that it is by no means the only plausible explanation. Each generation’s preference for mathematical economics could, for example, have been guided less by their ability to master the mathematical physics that they were taught and more by the templates of economics that were then dominant. It is clear that the first generation admired the clean logic of Mill’s narratives – they were the Cambridge Millites – whereas the second generation admired Marshall’s mathematical appendices – they were the first Cambridge Marshallians. It should never be forgotten that lesser men follow in the wake of great men as well as evolving institutions.

5 The Implications for Economics: The Importance of Relative Rankings

Historians of economics should, after reading Warwick’s study, also place greater emphasis on the relative wrangler positions of the graduates when tracing the lives of the Cambridge economists in the Victorian era. The professional advancement, additional life-time earnings and, above all else, prestige that came with an individual’s specific position in the order of merit explains why Marshall’s contemporaries invariably described him as both a second wrangler ***and*** a prominent economist. Indeed, it is sometimes uncertain which of these two descriptors his colleagues thought was more important. This explains why those who wrote the eulogies following Marshall’s death invariably added the rider, as I have in the previous sections of this paper, that he was second to Lord Rayleigh, one of the great mathematical physicists of the Victorian period and the Nobel Prize winner in Physics in 1904. In other words, by mapping out the extraordinary competition in the year of 1865, these contemporary commentators were further raising Marshall’s status as an intellectual. The players themselves were certainly concerned that their intellectual standing could suffer if the market-place failed to recognise that they had competed in an abnormally brilliant year. One can only imagine what the third wrangler in 1880 felt about his luck, when two of the great physicists of the late-Victorian age, J. Larmor and J.J. Thompson, were first and second wrangler respectively. Stephen caught this potential false market signal, and captured Fawcett's naked ambition, when he recalled that Fawcett 'used to say that he would rather be senior Wrangler in the worst year than second to Sir Isaac Newton. No man was more fully awake to the tangible commercial utility of a good degree' (Stephen 1885, p. 26; see also Moore 2002). Fawcett's observation also indicates, of course, that just as the worth of a candidate in the market-place could be underestimated in a brilliant year, it could also be overestimated in a mediocre year. Thus Russell's aforementioned assessment of Sanger's mathematical ability is somewhat tarnished, together with Russell's own reputation as a mathematician, by the fact that the examiner's report for that year included the observation that the quality of the answers of one individual only (the senior wrangler, a G.T. Manley) did not disappoint (Griffin and Lewis 1990, p. 61).

The desperate quest for a senior wrangler position also explains the early career moves of the individuals who are of interest to historians. For instance, the most 'glittering' of the Cambridge prizes to be won from a high wrangler position was a fellowship. At this time a fellow was, as a rule, elected from the college to which he belonged as an undergraduate and hence a wrangler would only gain a fellowship if demand did not outstrip the limited supply offered by a specific college. This fine example of J.E. Cairnes's theory of 'non-competing' groups caused great consternation amongst the prospective high wranglers, especially as the supply and demand imbalance differed markedly from college to college and from year to year. One would often have to come in the top five or so wranglers to a gain a position at an institution such as Trinity College, since it was wealthy, prestigious and had a long history of manufacturing senior wranglers, while one would need only be placed in the top twenty wranglers to gain a position at an institution such as Trinity Hall, since it was relatively modest, small and (until the 1860s) did not have a history of producing wranglers. Thus Marshall's second position in 1865 automatically secured him a fellowship at St John's, while Stephen needed only to gain a twentieth position to secure a fellowship at Trinity Hall. This characteristic of the fellowship market explains why prospective wranglers often migrated between colleges (sometimes with inducements from the host college in an attempt to increase its prestige). Fawcett migrated from Peterhouse to Trinity Hall, while Clarke migrated from Trinity College to Queen's College. Other senior wranglers merely bided their time until a position fell vacant. Flux, who was the son of journeyman cement-maker and therefore aware of the value of Tripos success more than most, did not join Marshall at St John's until 1889, two years after sitting his Tripos examinations. Still other senior wranglers simply missed out. Keynes junior, an individual with a strong grain of intellectual arrogance, failed to gain a fellowship on graduating because his twelfth position was deemed inadequate by King's College. This failure, in turn, induced him to sit the Civil Service examinations in 1906, for which he earned an equally disappointing second position that led to a position in the India Office instead of the Treasury.

The intense pressure to achieve a position in the top wranglers also explains why many young candidates baulked at the fence. To be placed amongst the top ten was seen as a triumph. To gain a rank between ten and twenty (like Keynes) was considered respectable, with some congratulating the candidate and others offering condolences (Harrod 1951, p. 104). To graduate as a senior or junior optime was deemed, for someone thought to be in the running, a complete failure. This last observation may partly explain why Stephen, Fawcett and the other wranglers looked down upon the mathematical economics presented by Macleod, a mere senior optime. Thus, fearful of failure and public humiliation, and lacking the required confidence, many simply chose not to play the game, while others broke down under the pressure and fled. I have always found it interesting that Francis Galton – the father of eugenics and the central figure in the movement in the 1870s to expel economics from Section F of the British Association – not only never had to 'struggle' for a living, but also withdrew himself from the chase for mathematical honours, becoming a pollman in 1844, when his mental health broke in the face of the competitive pressure. As already mentioned, the neurotic John Neville Keynes, lacking confidence and suffering from various stress-induced ailments, also withdrew from the race for mathematical honours, opting instead to pursue honours in the Moral Science Tripos. Keynes's tutor commented that he had too fine a mathematical mind to be wasted as a mere logician, but his (and Fawcett's) appeals to Keynes to stick with mathematics were to no avail. Keynes junior and Hawtrey, on the other hand, simply shunned the intense study patterns that were required for success and, spreading themselves thinly across many pursuits, paid the penalty with poorer than expected results. This led to the famous quip by Keynes's mathematical tutor at King's, who was no less than the aforementioned mathematical economist Berry, that young Maynard devoted all of his 'spare' time to the study of mathematics (Harrod 1951, p. 57). Although this stance is usually attributed to the new-fangled philosophies of life embraced by Keynes and Hawtrey, the ostentatious way in which these two paraded their lack of study also smacks a little of the scholarship man making excuses in advance of possible failure. Indeed, it was common throughout the age of the Tripos for many capable men to choose to go out as pollmen, turning to billiards or other cultural pursuits as the exams approached. The very phrases the students used indicate the common theme: 'funking fit', 'brain fever', the 'insomnia's miser'. It was these sorts of individual failures that induced a number of contemporary commentators to criticise the Mathematical Tripos examinations. These critics pointed out that not all scholars who had something to contribute possessed a temperament that was suited to such a competitive environment; that some great minds were unnecessarily narrowed by the wrangler machine; and that even inferior minds could sometimes excel within the narrow rules of the game. Stephen, in particular, became critical of tedious intellectual gymnastics and rather narrow interests displayed by his fellow wranglers.

6 Application to the History of Economic Thought: Manly Temper and New Liberalism

Warwick's extensive analysis of the athletic culture within the Mathematical Tripos should induce a reappraisal of the extent to which the manly temper of the Cambridge economists influenced their judgments on economic matters. The harder edges of the New Liberal philosophies espoused by the Cambridge economists (and which coloured all of their policy and theoretical writings) should, at least in part, be attributed to the competitive and manly culture generated by the Mathematical Tripos. As already indicated, this Tripos was just as much intended to test a candidate's weakness as it was a test of his ability (Warwick 2003, p. 216). The bookwork that each candidate needed to absorb required many hours of sustained labour and the marathon nature of the exam itself was deliberately designed as a test of endurance. Some candidates, admittedly, took to reading music and other cultural pursuits for the release from the mental strain that was required at critical junctures, but most took to some type of athletic pursuit. It was believed that such pursuits would *both* provide the required release from the intense mental outlays *and* build the character and stamina that was required to navigate the exam process. These sporting endeavours quickly developed into competitive events that mirrored the intense competition of the Tripos itself and what these young men presumed would be the mad scramble that they would face in adult life. The perfect intellectual was thereafter viewed as one who studied hard and rowed hard. This vision was reinforced as each generation reflected upon their undergraduate experiences through rose-tinted spectacles and praised (and indeed aided the escalation of) the athletic and exam culture. They believed that they had excelled on the field of play and in the examination hall through their own merits and, having succeeded, they expected others to pass through the same sort of ordeal. No one should receive a helping hand. It is therefore no wonder that the Cambridge economists of this age were suspicious of the state and largely restricted themselves to those New Liberal schemes in which the working classes could pull themselves up by their own bootstraps *if they so chose*. They, in short, may have embraced the ‘positive’ liberal act of providing the minimum level of endowments that were needed to allow agents both to make free choices (in the effective sense) and to fulfil their function, but they baulked at allowing the state actually to provide many of these endowments because of the way in which the resulting change in incentive structure would sap the agents’ energies.

The early Cambridge economists particularly embraced the cult of the athletic Stoic, who, confronted with inhuman intellectual toil and hindered by personal failings, manfully persevered. Whewell was a man of ‘splendid physical development’ who was a bold (or even reckless) rider and, to the astonishment of the more athletic undergraduates, would jump up the Hall steps in one leap. Legend has it that a famous prize-fighter of the day remarked of Whewell that a ‘bruiser was lost when that man became a parson’. Some of those who knew Whewell, such as Stephen, politely pointed out that this masculine vigour also gave him some unattractive qualities, such as an overbearing manner and impatience, but it was nonetheless admired by the Cambridge community (Stephen 1917, p. 1371; Trevelyan 1943, p. 97). He died after being thrown from a horse at the age of 72. Stephen himself became famous as a muscular scholar. He rowed, coached the Trinity Hall boat on the Cam, was a famous walker and became a noted mountaineer. He would think nothing of walking from Cambridge to London to attend a meeting of the Alpine Club; he was the first individual to climb the Schreckhorn; and he was the leader of the Sunday Tramps, a small walking group that included Edgeworth amongst its members in its fading years. The other members of Stephen’s set had a similar love of athletic pursuits: the blind Fawcett was a member of a veteran’s boat club called the Ancient Mariners and had a passion for skating, while Clarke eventually died from an inflammation brought on by excessive bicycling. All of them went on reading parties in the Lake District, where they climbed, hiked and attended wrestling matches during the day and discussed economics over games of whist in the evening (see Moore 2002). They also all had a healthy respect for those who manfully competed against the odds, particularly if it was clear that the candidate’s hand was insufficiently steady and his physical constitution ill-suited for the Tripos process. The senior wrangler to whom Courtney came second in 1855 was a ‘pale, rather sickly looking man with darkish hair.…rather narrow in chest, and with sloping shoulders' (Gooch 1920, p. 29). He went out for a walk not long after his success and was found dead in shallow ditch in a field about a mile from his college. The senior wrangler of Sidgwick's year suffered a mental breakdown after the exams of 1859, and on recovery discovered that he could no longer recall any higher mathematics (Groenewegen 1995, p. 87).

These and other tragic stories now make one's head turn away, but the Cambridge Millites revelled in them. The subjects of the stories may have somehow failed immediately on succeeding, but, in the eyes of their contemporaries, at least they 'played the game'. They did not funk it like Keynes junior or Harrod. Indeed, the later Cambridge economists who passed through the Mathematical Tripos on their way to their vocation never matched the manly and athletic culture of this early wave of migration from mathematics to economics. Marshall may have enjoyed rowing, but, contrary to the advice given to most candidates, his tutor discouraged him from this pursuit on the grounds that it might distract him from his studies. He instead gained release by light reading of Boswell's *Johnson* and became a famous hypochondriac (Groenewegen 1995, p. 83). Reference has also already been made to Keynes senior's withdrawal from the race for mathematical honours with depression and toothaches. One could only imagine how his avuncular confidant, the blind and stoically disposed Fawcett, viewed these excuses. Stephen, described by George Meredith as the 'Phoebus Apollo turned fasting friar', would certainly have little brook with such unmanly behaviour, even if he did find fault with the narrowness of the Tripos itself. Still, even though the later waves of migrants from the Mathematic Tripos did not match the athletic passions of the Stephen-Fawcett set, they still enjoyed outdoor pursuits and certainly did not escape the manly culture that then defined Cambridge. It was all-pervading. The unsociable Marshall may not have belonged to a boat club, but he liked rowing; Keynes senior enjoyed outdoor pursuits, such as cycling, like the next man; Sanger was a famous walker; and Bowley had a passion for cycling. In fact, Bowley would (along with Edwin Cannan) desperately cycle faster when riding with Edgeworth on the grounds that Edgeworth could not talk mathematics 'at more than eight miles an hour' (Allen and George 1957, p. 238). Even Keynes junior rowed, and played tennis, racquets and football. Indeed, if anything, the masculine temper was carried over into this later period to such an extent that the eventual success of the occasional woman in this masculine enterprise was instrumental in persuading many in Cambridge that women should be integrated within the New Liberal paradigm. For although women were not formally awarded degrees at this time, each unofficial success of a woman who sat the exams in the last part of the century (where their rank was expressed as $n+1$ and n was the formal male candidate) was taken as proof that they could compete with men on a level playing field. The placing of Henry Fawcett's daughter, Philippa Garret Fawcett, above the Senior Wrangler in 1890 probably did more for women's rights than any one suffragette meeting or feminist monograph. It was a national sensation (see McWilliams-Tullberg 1975, p. 102). She certainly upstaged her ambitious father, who in a small way funked his own exams due to anxiety-induced insomnia (see Stephen 1885, p. 32; Warwick 2003, p. 187). The notion of the heroic intellectual-athlete was only challenged, but not completely displaced, with the rise of the athletic hearties in the 1900s and hence the association of physical exercise, especially by those

Apostles who embraced what became known as the higher sodomy (Deacon 1985, p. 55), with anti-intellectualism.

7 Application to the History of Economic Thought: Common Coaches and the Cambridge Style

Warwick's analysis of the coaching system within the Mathematical Tripos also sheds important light on the way in which economics developed at Cambridge. Just as each violinist displays a distinctive style of playing that is partly inherited from a master, each wrangler displayed a mathematical style partly inherited from his coach (Warwick 2003, p. 244). Furthermore, because all coaches themselves had similar coaching lineages, there was, overall, a distinctive 'Cambridge style' of solving applied mathematical problems. This observation, by the way, gives rise to some unexpected insights into the issue of the extent to which Cambridge and non-Cambridge economists – such as Marshall and Jevons – received different training in mathematics. On the one hand, it is clear that non-Cambridge economists rarely received the rigorous training in mathematics over a sustained period that the Cambridge economists received; but, on the other hand, they were increasingly trained by exported Cambridge men who had inherited their coaching style from the complex Cambridge coaching lineage. For example, Jevons was trained in mathematics for two years at University College by Augustus De Morgan, who had been bred in the Cambridge factory and had gained the title of fourth wrangler in 1826. De Morgan had earlier trained Routh (senior wrangler 1854) at University College before the latter went up to Cambridge. Routh then had the Cambridge mathematics style reinforced by the Cambridge coaching system, first via Todhunter and then via Hopkins. Routh, in turn, went on to train Marshall, who was himself prepared at school by recent wrangler, J.A.L. Airy (second wrangler 1846), who was also almost certainly a pupil of Hopkins. In other words, Marshall and Jevons were, to some extent at least, trained by the same family of coaches. Just as the doctoral graduates in economics from the Chicago factories of the 1960s and 1970s imposed the subtle (and often not so subtle) Chicago style throughout the economic departments of the world, the Tripos men eventually imposed the Cambridge vision of mathematical physics on the institutions of the British Empire. By the 1860s nearly all of the mathematics professors at Birmingham, Manchester and London were senior wranglers, while many schools (such as Merchant Taylor's School in London, where Airy trained Marshall: see Warwick 2003, p. 257) recruited recent senior wranglers as schoolmasters and transformed themselves into what eventually became known as 'wrangler nurseries'.

The issue of the extent to which the mathematical training of Victorian economists differed has simply not been adequately addressed in the literature. Although one could draw a rather long bow between the training of Marshall and Jevons (if only to encourage someone to investigate and debunk it), there is no evidence of a Cambridge influence on Edgeworth (who was initially coached by John Kells Ingram, the geometer-cum-economist-cum-everything from Trinity College, Dublin) nor on Henry Charles Fleeming Jenkin (who studied physics at Genoa University before being apprenticed as a mechanical engineer in Manchester). A systematic study of these differences and similarities in mathematical training would certainly be an important contribution to the ever-growing literature on the controversial hypotheses advanced by Philip Mirowski, in works such as *More Heat than Light* (1989), that the first neoclassical economists simply replaced energy with utility and then brazenly purloined the constrained-optimisation approach that was employed in energetics for use in economics. Readers may be relieved that, at this point, I have no intention of making yet another contribution to this once thought-provoking but now rather tired debate within the tight compass of this critical review. There will be no death by a thousand references to energetics. I will, however, point out that Warwick's research indirectly highlights a glaring fault in Mirowski's historiographical approach, namely, the lack of institutional detail (relating to the Mathematical Tripos and other institutions) that would explain the way in which the energetics metaphor was imported into (and sustained in) the economic arena. Mirowski has tried to make his case by providing internal readings of one or two 'big' texts, such as those written by Jevons and Walras, rather than via the far more arduous 'science studies' approach (as exemplified by Warwick) of reconstructing the institutional framework in which such ideas actually evolve. This is, of course, one of the main reasons why many historians of thought have yet to be convinced by his arguments (see, however, Mike White's (2004) more nuanced prize-winning essay on the subject).

Indeed, instead of attempting to trace the way in which the Cambridge economists transplanted a particular theory, model or lemma from the domain of mathematical physics to the domain of economics, I wish to end this review by suggesting one or two ways in which the methodological style adopted by the Cambridge economists was influenced by their early training in the Mathematical Tripos.

First, Cambridge coaches made sure that their charges were drilled in what Warwick calls the principle of 'Technical Unity'; that is, the 'analogical application of common mathematical methods' (and common physical principles) to solve problems from a range of different physical domains. This principle was seen as being extremely important for exam preparation in the field of applied mathematics, since it enabled students to comprehend and solve problems in a range of physical settings with which they were, at first blush, unfamiliar. Routh and Webb, in particular, devoted a great deal of time to ensuring that their charges understood the solution of the Laplace equation, since the concept of a potential could be deployed to solve problems relating to gravitational theory, thermodynamics, electrostatics, magnetostatics and electromagnetism (Warwick 2003, p. 278). Lagrangian dynamics and the principle of least action were similarly focused upon in coaching sessions as they too could be applied in a range of situations. This close-order drill during exam preparation also became important for actual research in applied mathematics at Cambridge. When, for example, 'wranglers came across new methods for tackling problems in fields such as acoustics and hydrodynamics, they naturally assumed that the same methods would find useful application in, say, thermodynamics or electromagnetic theory' (Warwick 2003, p. 278). It is safe to speculate, then, that the Cambridge economists who were so drilled in their undergraduate days yearned for similar engines (not necessarily mathematical) that could solve problems from a range of social domains that interested them in their mature years (such as production and different industries within production, consumption and different types of consumption, public finance, and so on). This approach – which economists accept without question today, but which was very nearly rejected in the Victorian period after coming under attack by the English historical economists (see Moore 1999, 2003) –

partly (but not wholly) explains Marshall's claim that economics is defined by the study of the 'one in the many and many in the one'.

Second, although the Mathematical Tripos was specifically designed to train students to think for themselves (in the great liberal arts tradition) by forcing them to solve a series of applied mathematical problems that were firmly grounded in the concrete world, the problem sets themselves became increasingly fantastic and unreal as the difficulty of the Tripos examinations escalated throughout the Victorian period. The problem sets were regularly lampooned for this reason (see Roth 1971, p. 230). At the same time, the increasing amount of bookwork that students needed to master to succeed in the final examination ensured that they often did not have time to gain an intuitive insight into the physical events that were being explained by their mathematical manipulations. It was easier simply to comprehend the way in which each step followed another in a logical mathematical manner. Indeed, sometimes candidates simply skipped or memorised entire sub-fields in the quest to gain ground before the exam. G.H. Hardy, for one, began to wonder whether applied mathematics at Cambridge was quite so perfect when Russell (who often struggled in his Tripos preparation) informed him after three years of study that he had not heard of Maxwell's equations (Griffin and Lewis 1990, p. 62). This contradictory situation of a tradition of an applied mathematics firmly grounded in reality, together with the misapplication of this tradition with the setting of increasingly bizarre and difficult problem sets that needed to be comprehended and solved at pace, moulded the Cambridge attitude to mathematical economics. This attitude, which is often mistakenly interpreted as an outright hostility to the application of mathematics to economics, entailed the belief that mathematics should be the servant rather than the master of economics and that the use of pseudo-mathematics (of the showy, intellectual gymnastics sort) should be condemned outright (see O'Donnell 1989, chapter 9). It is clear that the applied tradition of the Mathematical Tripos meant that the Cambridge economists had little time for (or even comprehension of) the formal proofs of pure mathematics, while the abuse of the applied tradition, which they had experienced first-hand, meant that they had little time for any mathematics if it got in the way of the intuitive understanding of the problem. These attitudes partly explain Marshall's famous advice to Bowley to 'burn the mathematics' if it could not be translated into English and his own practice of confining mathematical economics to an appendix. Such attitudes are also reflected in many passages from Maynard Keynes's writings, such as the rather revealing comment from his 1907 probability dissertation:

> Every mathematical student knows well the sensation of being pushed on by an eager coach into the intricacies of the analysis of a new subject before he really knows what on earth it is all about. At first he desperately endeavours to understand what each equation means; but soon he is swept off his feet and must content himself with the easier task of perceiving how each successive step in the algebra follows from those which precede it. (This is cited in O'Donnell 1989, p. 187, who, I should add, correctly points out that many of these passages can also be attributed to Keynes's wider philosophy. See also the references to pseudo-mathematics in Keynes 1936, pp. 275, 297-8).

In any event, this interpretation of Marshall's (and Keynes's) hostility to a certain type of mathematical economics is in some ways similar to the one advanced by Weintraub (1985, p. 167; 1997, p. 2044; 2002; 2004), but, unlike Weintraub's hypothesis, it does not include his more speculative rider that Marshall's position was also governed by Marshall's inability to keep up with the ever-changing nature of mathematics at Cambridge and elsewhere.

Third, students within the Mathematical Tripos were encouraged to render their solutions to the applied mathematical problems in diagrammatic form. Those who championed the liberal arts interpretation of this degree preferred students to employ geometric reasoning rather than the (French!) symbolic reasoning. As Whewell argued, with geometric reasoning 'we tread the ground ourselves, at every step feeling ourselves firm, and directing our steps to the end aimed at', but, with analytical reasoning, 'we are carried along in a railroad carriage, entering in at one station and coming out of it at another, without having any choice in our progress in the intermediate space' (which is an argument not dissimilar to that advanced by Keynes in the previous paragraph). Whewell argued that the latter method was an appropriate way for a businessman to travel, but it was not fit for the 'gymnastics of education' (quoted in Warwick 2003, p. 102). The Cambridge preference for visual representations of problems is also clearly revealed in some of the answers to Tripos questions that have been reproduced by Warwick. It is, I believe, this preference that explains why those Cambridge men who migrated to the social sciences were so ready to represent their ideas diagrammatically (with Keynes being a famous exception). Stephen represented the laws of demand and supply diagrammatically in the early 1860s, Venn reconstructed Boole's logical processes diagrammatically, Marshall found it entirely normal to translate passages from Mill's *Political Economy* into relationships in two-dimensional space, and Bowley constructed extremely messy contract curves in the Edgeworth-Bowley box.

These themes should, of course, be elaborated at far greater length, and many other methods of inquiry could be traced to the arcane drills and customs of the Mathematical Tripos. It is sufficient in this review essay to state that any account of the Cambridge style of treating economics must hereafter include references to Cambridge mathematical physics.

8 Conclusion

The Mathematical Tripos was the 'most difficult test that the world has ever known, one to which no university of the present day can show any parallel' (Roth 1971, p. 228). It produced the greats of Cambridge mathematical physics, including Stokes, Kelvin, Clerk Maxwell, Rayleigh, Larmor and Thomson. It produced several generations of philosophers with an extremely good knowledge of mathematics and physics, ranging from Whewell and Stephen to Sidgwick and Russell. In the liberal arts tradition, it produced endless cohorts of men (with great problem-solving abilities) for the professions and administrative positions throughout the Empire. It even managed to throw up some pure mathematicians, in spite of the hostility shown towards this breed at Cambridge, such as Cayley and Hardy. But, what is of most importance to the readers of this journal, it produced nearly all of the Cambridge economists who dominated the discipline of economics in the late nineteenth century and early twentieth century. In short, although the Mathematical Tripos had many obvious failings, and although many worthy candidates were either ruined by its excesses or forced down wrong avenues of research by its arcane and often outdated curriculum, one should never underestimate its influence within the Victorian world of ideas. Warwick has provided a great service by making this fact clear in *Masters of Theory*. He has also inadvertently exposed a huge gap in the literature relating to the history of Victorian economic ideas. It is, to say the least, puzzling that no scholar (*pace* Groenewegen 1995 and Weintraub 2002) has presented a *systematic* study of the links between mathematical physics and economics at Cambridge. Yet now the road lies ahead for those scholars who wish to write something more than a review essay. Historians of economic thought interested in this period of intellectual history may take up any number of lines of research from the base provided by Warwick. On the other hand, reading a 572-page book on mathematical physics may be a little too time-consuming for those who have only a passing interest in this issue. I suggest that these scholars turn to the slim, highly amusing and posthumously published essay on the Mathematical Tripos by the geometer, Leonard Roth (1971). It is the best of its type and, in particular, it provides a nice counter-balance to Weintraub's (2002, chapter 1) rather jaundiced, but nonetheless striking, half chapter on the subject.

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Notes

1 I am grateful to John King, Mark Donoghue, Ray Petridis, Mike White and several anonymous referees for reading drafts of this review essay.

2 One could also include a number of marginal players in this group. The most likely would be Henry Fountain, who was twenty-sixth wrangler in 1892. Although he only took Part I of the MST (apparently by mistake), Marshall reported that even then he knew twice as much as those who took Part II. Fountain came second to Bowley in the Adam Smith Prize in 1894 (M. to Eleanor Sidgwick, 16 February 1896; Whitaker 1996, volume 2, p. 159). W.E. Johnson could also be included as a major player. Although he was slightly older, he interacted with members of this group. He co-authored with Sanger a paper on the mathematical theory of demand that was presented to the Cambridge Economics Club in 1894. He also published an article on the 'Pure Theory of Utility Curves' in the *Economic Journal* in 1913, which, Keynes argued, carried the application of mathematical analysis to economic theory about as far as it was likely to be useful to go.

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