Cournot on Trade Between Regions and the Transition from Partial to General Equilibrium Modelling

John Creedy*

I. Introduction

This article examines Cournot's analysis of trade between two regions, contained in his famous *Researches* (1838, translated 1927). Cournot's framework concentrated on the market supply and demand for a single good and the effect of allowing trade to take place between two regions that were previously isolated[1]. A reason for considering Cournot's model is that it was a major influence on both Marshall and Walras. Both of these authors criticised Cournot's analysis because its partial equilibrium concentration on a single good meant that it could not be applied to the analysis of *exchange*. Walras provided a direct extension of Cournot's model, even retaining the original notation, while Marshall produced his diagrammatic analysis of trade that is essentially the same model as that produced by Walras. An understanding of Cournot's model therefore helps to see the related work of Marshall and Walras in a clearer perspective.

Cournot's framework is described in Section 2, along with a diagrammatic version. The fundamental criticism raised by Walras and Marshall, that the partial equilibrium version is not appropriate for the analysis of exchange, is discussed in Section 3. This section shows how the partial model can be extended to a general equilibrium model. Section 4 briefly concludes.

2. The Basic Model

The framework

Cournot considers a single good produced and consumed in two separate markets A and B. The market demand and supply curves are taken by Cournot as given, and the regions have a common currency. In isolation the equilibrium price of the good is p_a and p_b in markets A and B respectively, with demand functions $F_a(p)$ and $F_b(p)$,

and supply functions $\Omega_a(p)$ and $\Omega_b(p)$. The prices are given by the intersecting partial equilibrium curves and are the solutions to:

$$\Omega_a(p_a) = F_a(p_a) \tag{1}$$

$$\text{and } \Omega_b(p_b) = F_b(p_b) \tag{2}$$

If $p_a < p_b$ and the difference exceeds the cost of transporting the good between the two markets, ε, then the good is exported from A to B. Cournot argues that trade equalises the price of the good in the two markets, except for the transport costs. If the new equilibrium price in market A is denoted p'_a, Cournot (1927, p.119) states that this is given as the solution to:

$$\Omega_a(p'a) + \Omega_b(p'a + \varepsilon) = F_a(p'a) + F_b(p'a + \varepsilon) \tag{3}$$

Which simply states that total supply is equal to total demand in both markets combined. Cournot then writes:

$$p_a' = p_a + \delta \text{ and } p_b = p_a + \omega \tag{4}$$

so that δ is the change in the price in market A and ω is the pre-trade absolute difference between prices in the two markets. Trade takes place only if $\omega > \varepsilon$. Substitute for $p_a = p_b - \omega$ in the first of the expressions in (4) and add ε to get:

$$p'a + \varepsilon = p_b + \delta + \varepsilon - \omega \tag{5}$$

Equation (3) can then be re-written as:

$$\Omega_a(p_a + \delta) + \Omega_b(p_b + \delta + \varepsilon - \omega) = F_a(p_a + \delta) + F_b(p_b + \delta + \varepsilon - \omega) \tag{6}$$

This expression can be simplified using Cournot's method of 'development and reduction' which involves taking the Taylor series expansion of each function of the form $F(p+\delta)$ and neglecting squares and higher powers of δ. Thus:

$$F(p + \delta) = F(p) + \delta F'(p) \tag{7}$$

Expanding each term in (6) in this way, and using (1) and (2), Cournot (1927, p.120) obtained:

$$\delta\{\Omega_a'(p_a) - F'_a(p_a)\} = (\delta + \varepsilon - \omega)\{F_b'(p_b) - \Omega_b'(p_b)\} \tag{8}$$

Since the market demand curves are assumed to slope downwards and market supply curves to slope upwards, the term in curly brackets on the left hand side of

(8) is positive, while that on the right hand side is negative. Since $\delta > 0$, then $\delta + \varepsilon - \omega < 0$ and $\delta < \varepsilon$. Hence the increase in price in market A must be less than the difference between the initial price differential and the unit transport cost.

Cournot went on to examine the conditions under which the total demand in the two markets combined would increase, which can be seen to depend on the ratio of demand to supply elasticities in each country (though of course Cournot did not use the concept of elasticity himself). He also considered the question of whether the value of output would increase. In attempting to allow for import or export taxes, Cournot made an algebraic slip which led him to believe that the price may fall in the importing country (although in fact price must rise in the importing, and fall in the exporting, country). This was briefly discussed by Edgeworth (1894, reprinted in 1925, ii, p. 49), where it is significant that he noted that Berry and Sanger, two former pupils of Marshall, had independently made the correction. The error was later also pointed out by Fisher, in Cournot (1927, p. xxiv).

A diagrammatic version

Cournot's framework was a relatively simple one in which a single good is initially produced in two countries that are isolated from each other. When 'communication' between the markets occurs, the good is produced and exported by the country in which it is initially cheaper, allowing for transport costs. The equilibrium requirement is that the supply (in the exporting country) is equal, at the new price, to the aggregate demand of both countries combined. Although Marshall published nothing on this analysis, some early notes are reproduced in Marshall (1975, II, pp. 246–248), which show his early attempt to cast Cournot's model into diagrammatic form, mainly for the purpose of examining the gains from trade using measures of producers' and consumers' surplus.

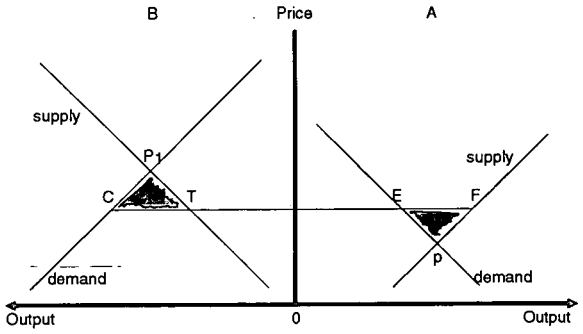

Figure 1 Cournot's Model

The diagrammatic analysis of Cournot's model was later refined by Marshall's former student Henry Cunynghame (1892, 1903), who argued that 'the method of treating economics graphically is probably due to Cournot' and added, 'the chief credit of reviving an interest in this method rests with Professor Marshall' (1892, p.36). After an unsatisfactory start (1892, p. 44), Cunynghame produced a 'back to back' diagram without any reference to Cournot but virtually paraphrasing the latter's introduction to his model (1903, p. 317). It does not seem to be widely recognised that Cunynghame's treatment stems directly from Cournot. Even Viner (1955, pp. 589), who refers to Barone's use of the same diagram to measure the gains from trade, does not seem to recognise that the diagram represents Cournot's model. The origins are, however, recognised by Samuelson (1952).

Ignoring transport costs, the diagram is shown in Figure 1 where the equilibrium price is such that CT = EF. Marshall's notes on Cournot show clearly the influence on Marshall's analysis of consumers' and producers' surplus. Marshall's diagrams translate Cournot's surplus analysis (of chapter 12) into the now familiar triangles. Using the back to back version of Figure 1 the left hand side shows that the gains to B's consumers arising from the price reduction outweighs the loss to producers, so that the net gain is equal to the area P_1CT. The price increase in A produces a net gain equal to the area EPF in the right hand side of the figure. Marshall added that if in each country the cost of production is independent of output, then the exporting country gains nothing from trade (1975, II, pp. 2478). Although these notes are not dated, there seems little doubt that Cournot was the sole influence and that Jenkin's (1871) analysis was quite independent, as Marshall himself always insisted.

3. A General Equilibrium Version

A fundamental criticism of Cournot's model is that it deals with only one good. This point is acknowledged by Cournot towards the end of the *Researches*, where he writes that,

> 'It will be said that it is impossible for exportation of a commodity to fail to involve importation on the exporting market of a precisely equal value; and reciprocally, importation on a market involves exportation of an equal value . . . It would be necessary to consider each of these nations as acting simultaneously the part of an importing nation and that of an exporting nation, which would greatly complicate the question and lead to a complex result' (1927, pp.161-162).

Cournot did not, however, pursue this question. It is more complex than simply adding another good and imposing a balance of payments constraint. Additional partial equilibrium demand and supply curves cannot by their very nature cope with the interdependence which is at the heart of the problem. However, Cournot's model can form the basis of a general equilibrium approach, as shown in the following subsection.

A simple extension

Suppose that there are two goods, X and Y, and comparative advantage is such that country A exports good X to country B, while the latter exports good Y to A. Assume complete specialisation, and denote the relative price of good X as p. This relative price can be interpreted as the amount of good Y that must be given in order to obtain a unit of good X. For present purposes it is necessary to express B's demand for X and A's demand for Y as $F_b(p)$ and $F_a(p^{-1})$ respectively; p^{-1} is of course the relative price of Y. The essential feature of an exchange model is that the demand for one good, at a given price, automatically carries with it a supply of the other good. B's supply of Y, corresponding to the demand $F_b(p)$, is thus given by:

$$\Omega_b(p) = p\,F_b(p) \tag{9}$$

while A's supply of X is given by:

$$\Omega_a(p) = p^{-1} F_a(p^{-1}) \tag{10}$$

The equilibrium price is that value of p for which the demand for and supply of, say Y, are equal. This requires:

$$\Omega_b(p) = pF_b(p) = F_a(p^{-1}) \tag{11}$$

which is equivalent to the equilibrium condition for good X, given by:

$$\Omega_a(p) = p^{-1}\,F_a(p^{-1}) = F_b(p) \tag{12}$$

The general equilibrium model requires only the specification of the two demand functions in terms of the relative price, p. This important insight was also fully recognised by Whewell when he constructed a mathematical version of J.S. Mill's trade model; see Creedy (1989).

Linear demands

In order to explore the nature of the model, suppose that these functions are linear, such that:

$$F_b(p) = a - bp \tag{13}$$

and

$$F_a(p^{-1}) = \alpha - \beta p^{-1} \tag{14}$$

From A's demand for Y in (14), the corresponding supply of X is obtained using (10) as:

$$\Omega_a(p) = \alpha p^{-1} - \beta p^{-2} \tag{15}$$

and equilibrium price is that which equates (15) and (13), giving:

$$\beta - \alpha p + a p^2 - b p^3 = 0 \tag{16}$$

so that three equilibria, not necessarily real or distinct, will exist. This approach therefore rapidly gives rise to the need to consider the stability of alternative equilibrium positions. The comparative static properties of models with multiple equilibria are of much interest, since small changes in demand conditions can lead to a large jump in the equilibrium price.

Notice that the supply curve of X is 'backward bending' (if p is on the vertical axis), with supply reaching a maximum when $p = 2\beta/\alpha$ and a point of inflexion where $p = 3\beta/\alpha$. Furthermore the maximum supply, where the price elasticity of supply is zero, occurs at a price for which the elasticity of demand (for Y) is minus one. It is the backward bending property that gives rise to the possibility of three equilibria. Hence a necessary condition for multiple equilibria is that the demand curve for one of the goods has an elastic range.

The analysis may be extended by using (13) to write $p = \{a - F_b(p)\}/b$. Substituting this expression for p into equation (9) gives:

$$\Omega_b(p) = F_b(p)\{a - F_b(p)\}/b \tag{17}$$

Equation (17) has a simple interpretation as the 'offer curve' of country B. This offer curve is clearly quadratic, so that if both countries have linear demand curves, the offer curves may intersect three times, consistent with the result from (16). It is well known that the turning point of an offer curve occurs at the point of unit demand elasticity.

Walras and Marshall

The previous subsection has provided what seems to be the most direct way of extending Cournot's model to deal with exchange. What is required is the recognition of the 'reciprocal' nature of supply and demand, combined with the appropriate interpretation of the relative price and the use of this relative price in specifying demand functions. Although this represents a small analytical step, it provides a major transformation of Cournot's model and its exploitation has an interesting history.

The concept of reciprocal demand combined with the clear idea of demand as a schedule was in fact explored by John Stuart Mill almost a decade before Cournot's book was published, although Mill did not publish his analysis of international trade until 1844. A mathematical analysis of Mill's model was produced by Whewell in 1850, but his approach was limited by his rather awkward specification of the demand functions directly in terms of elasticity; for a detailed analysis see Creedy (1989).

The above analysis has, however, produced a model which is essentially the same as that of Walras (1874). Furthermore, a minor extension was seen to produce offer curves, which were of course the focal point of Marshall's analysis of international trade. This analysis was published privately by Sidgwick in 1879 (reprinted in

Marshall, 1975), but had to wait over forty years before a selection was published by Marshall (1923). The fundamental similarity of the basic models used by Walras and Marshall, who both stressed multiple equilibria and examined stability properties, is clear although some economists, including Jaffé (in Walras, 1954, p. 504,) have argued that the approaches were different. The 'substantially equivalent' nature of the two analyses were also stressed by Hicks (1934) in his brief but penetrating review of Walras. Hicks considered the question of whether the simultaneous development of 'a very new line of thought' was related to the nature of the subject and the quality of the contributors. He suggested that, 'One feels almost obliged to explain it by the intrinsic excellence of the path they followed ... Yet in fact there is a clear historical reason for it, one decisive influence we know to have been felt by both. Each of them had read Cournot' (1934, reprinted in 1964, p. 383).

This question of the filiation of ideas, raised by Hicks, is also of interest. Marshall acknowledged a general debt to Cournot, and in considering the analysis of trade, he explicitly rejected Cournot's treatment because of its partial equilibrium nature. This was apparent in his first published paper in economics, where Marshall indicated his preference for the general approach of Mill rather than Cournot (See Marshall, 1876). Marshall's subsequent statement that his trade diagrams were 'set to a definite tune, that called by Mill' also confirms the importance of Mill. It is well known that Marshall, with his highly polished style, usually concealed his own path taken (particularly his mathematics) to reach his discoveries and it is unfortunate that his process of arriving at the concept of offer curves remains unknown. What is surprising in this context is that Marshall does not seem to have been aware of Whewell's extensive analysis. Indeed, it would not be inaccurate to describe the above extension of Cournot's model as a Mill/Whewell/Walras/Marshall model, the development of which has been far from 'linear'.

Walras's approach does, however, appear to have been a direct modification of Cournot's model along the lines shown above, although Walras's 'exuberance of algebraic foliage' and his awkward diagrams make his analysis less than transparent. This seems to be the case despite Walras's thorough familiarity with Mill's work, and his published correspondence suggests that he did not become aware of Whewell's model until after he made contact with Jevons; see Jaffé (1965, I, letters 328, 375).

Walras's autobiography clearly states that he 'soon perceived' that Cournot's approach could not be applied to exchange and: 'Restricting my attention, therefore, to the case of two commodities, I rationally derived from the demand curve of each commodity the supply curve of the other and demonstrated how current equilibrium results from the intersection of the supply and demand curves. Then I proceeded to derive the demand curve itself from the quantities possessed by each individual in the market and from each individual's utility curves for the two commodities considered' (quoted in Jaffé, 1983, p. 25). Walras's transformation of Cournot's model is contained in the *Elements* (1954, pp. 81-114), where it can be seen that he even retained the use of Cournot's notation. The crucial ingredient is, in Walras's terms, the recognition that 'to say that a quantity D_a of (A) is demanded at the price p_a is, *ipso facto*, the same thing as saying that a quantity D_b of (B), equal to $D_a p_a$, is being offered' (1954, p. 88).

In considering the genesis of Walras's analysis, Jaffé (1983, pp. 5577) has argued that there is a direct line of filiation from Isnard to Walras. Isnard recognised the

important point that the price ratio is equivalent to the (inverse) ratio of quantities exchanged, which as stressed above is crucial for the theory of reciprocal demand. He also addressed the mutual interdependence in a general equilibrium system. But Isnard was not alone here, and his discussion is restricted merely to given quantities; there is no analysis of demand as a function of relative prices. A section from Isnard's analysis is reproduced in Baumol and Goldfeld (1968, pp. 255-257), who also suggest that strong claims made for him are 'somewhat over-enthusiastic' (1968, p. 253).

Although Cournot's model was limited to a partial equilibrium approach, he nevertheless had a clear view of the importance, and difficulty, of general equilibrium analysis. The following statement shows his insight into the issues:

> in reality the economic system is a whole of which all the parts are connected and react on each other. An increase in the income of the producers of commodity A will affect the demand for commodities B, C, etc., and the incomes of their producers, and, by its reaction, will involve a change in the demand for commodity A. It seems therefore, as if, for a complete and rigourous solution of the problems relative to some parts of the economic system, it were indispensable to take the entire system into consideration. But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically. (1927. p. 127).

The problems were indeed to surpass the powers of analysis until the middle of the twentieth century, and the methods of calculation until more recently. But Walras concentrated on stating the structure or *form* of the general equilibrium model, without attempting to specify the precise nature of the equations and solve them. In so doing, the influence of Cournot has been seen to be vital.

4. Conclusion

When discussing Cournot's contribution to trade theory, Edgeworth commented, not without sympathy, that: 'The lesson of caution in dealing with a subject and method so difficult is taught by no example more impressively than by that of Cournot. This superior intelligence . . . seems not only to have slipped at several steps, but even to have taken a wholly wrong direction.' (1925, ii, p.47). Cournot's analysis is indeed severely limited by its partial equilibrium framework. Nevertheless, his model provided an influential starting point for the development of a general equilibrium approach. A major value of his work seems to have been the stimulus it provided to Walras and Marshall to attempt to improve the basic model. By producing a general equilibrium version of Cournot's model, the present article has hopefully shown that the analyses of later generations can be seen in a clearer light.

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* Department of Economics, University of Melbourne, Parkville, Vic. 3052

Note

1.This chapter of Cournot's has been largely neglected in the literature on the history of international trade theory. It was criticised briefly by Edgeworth (1894, reprinted 1925), and was not considered in Chipman's (1965) survey. While Viner (1955) provided a detailed criticism of Cournot's treatment of the gains from trade (involving consumers' and producers' surplus), he did not examine the trade model.

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