

Lewis Carroll, Political Economist

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Duncan Black (1970, 28) refers to Lewis Carroll's work on voting theory as 'the most distinguished contribution that has been made to Political Science since the seventeenth century'. Due to Black's (1958) clear explication of their pioneering achievements, Carroll's 1873, 1874, and 1876 pamphlets on the properties of different election rules, vote cycling and 'disequity' are widely recognized. Black (1967, 1969, 1970) discussed Carroll's later letters and pamphlet (*Principles of Parliamentary Representation*, 1884) on other voting issues. Carroll clearly holds an important place in the tradition of choice theory which entered economics with Arrow (1951, 1952). Less well known, however, is the economic content of other works published by Dodgson under his better-known pseudonym, Lewis Carroll¹. The briefer remarks on political economy embedded in *Sylvie and Bruno* (1889) and *Sylvie and Bruno Concluded* (1893) which this paper first addresses are perhaps of minor interest despite their appearances on the back cover of the *Journal of Political Economy* over the years. However, I also cite Carroll's more serious essays in economics. This paper is an attempt at completely recording and discussing passages of economic content in Carroll's work other than those in public choice.

John Maynard Keynes wrote to Harold Macmillan in a reader's report on what was to become Joan Robinson's *Economics of Imperfect Competition* that he suggested removal of a quote from *Sylvie and Bruno* because 'Dennis Robertson really must be considered to have a patent in quotation from Lewis Carroll in economic works' (JMK, XII, 867), in which he used them as chapter heads. Robertson chose quotations not for economic content, however, but as amusing illustrations of or counterpoints to the principles he discussed. In *Money* (1928), Robertson quotes only from *Alice's Adventures in Wonderland* and *Through the Looking-Glass and What Alice Found There*. Similarly, econometricians have used chapter heads from *The Hunting of the Snark*. Curiously enough, these are Carroll works which seem to me to have no economic content whatsoever, while *Sylvie and Bruno* (S&B) and still more *Sylvie and Bruno Concluded* (S&BC) contain quite a lot of economics.

It may be that just as fame made Carroll feel free to pontificate on vivisection, child labour in theatres, and children's religious education, it caused him to think his ideas about political economy of general interest. He was not entirely wrong. His

best work was certainly in the area of public choice and voting theory— not surprising for a mathematician whose natural bent was toward logic. In this then-undeveloped area his ideas were pathbreaking. He also (and, I think, quite unconsciously) made an incredibly early mention of the characteristics analysis of services, although it is marred by a strong implicit assumption about preferences.

Carroll's other comments on political economy fall into two categories: *jeux d'esprit* and short, but serious, considerations of questions of Christian political economy. At least one of his funny economics stories is also a nice passage on bilateral monopoly. Carroll's ventures in such topics as the morality of property rights over inherited possessions were undistinguished, however, probably partly because he had read little or no political economy. Carroll's discussion of equitable procedure in lawn tennis tournaments uses a methodology similar to that of his axiomatic approach to the properties of various voting schemes, but actually addresses a very different problem.

Outlandish Economics

All but one of Carroll's deliberate expositions of ludicrous economic propositions is limited to the political economics of a country named Outland, on the border of Elfland. Its ruler is called the Warden², and his regent is the Sub-Warden, which might seem to suggest that the ridiculous political economy of the nation is a deliberate mockery of academic economics. Some of the jokes are simple. A hired, but poorly rehearsed, crowd accidentally cries 'Hoo-roah! Noo! Consti! Tooshun! Less! Bread! More! Taxes!' (S&B, 387). Outlanders took what Carroll called the English Theory of Political Dichotomy a bit too far: the Outlander Mein Herr describes it as a system in which "...the function of the 'Ins' ... was to do the best they could for national welfare— in such things as making war or peace, commercial treaties, and so forth[.]... And the function of the 'Outs' was... to *prevent* the 'Ins' from succeeding in any of these things[.]" In extension of this principle, in Outland they hired doers and undoers in agricultural production, but found this unsatisfactory (S&BC, 619-622).

The people of Outland have a few resources that we do not, however: Mein Herr says that "...in a country I have visited, ... they store [wasted time] up: and it comes in *very* useful, years afterwards! For example, suppose you had a long tedious evening before you: nobody to talk to: nothing you care to do: and yet hours too soon to go to bed.... When that happens to— to the people I have visited,... they store up the useless hours: and on some *other* occasion, when they happen to *need* extra time, they get them out again." (S&BC, 584).

Unfortunately Mein Herr can't explain the process of storage in English, and his auditors don't understand Outlandish.

Some of Carroll's sallies are rather more sophisticated. The Professor (who appears in our world as Mein Herr) puts off his tailor by means of an endless Ponzi game:

"How much is it this year, my man?" ...

"Well, it's been a doubling so many years, you see,"

the tailor replied, a little gruffly, "and I think I'd like the money now. It's two

thousand pound, it is!"

"Oh, that's nothing!" the Professor carelessly remarked.... "But wouldn't you like to wait just another year, and make it *four* thousand?..."

"...it *dew* sound a powerful sight o' money! Well, I think I'll wait—"

...."Will you ever have to pay him that four thousand pounds?" Sylvie asked....

"*Never*, my child!... He'll go on doubling it, till he dies. You see it's *always* worth while waiting another year, to get twice as much money!" (S&B, 432-433).

Monetary policy and public reactions to it are also a bit odd in Outland. The Professor

"...had to be present at the Election, you know, as the author of the new Money-Act. The Emperor was so kind as to wish that *I* should have the credit of it.... [Nonetheless] the Emperor started the thing.... He wanted to make everyone in Outland twice as rich as he was before— just to make the new Government popular. Only there wasn't nearly enough money in the Treasury to do it. So I suggested that he might do it by doubling the value of every coin and bank-note in Outland.... And you never saw such universal joy.... Everyone's buying everything!" (S&B, 502).

An ancient manuscript read by Mein Herr is Outlandish, but has rich economic content indicating Carroll's awareness of the problem of bilateral monopoly.

"In a city that stands in the very centre of Africa, and is rarely visited by the casual tourist, the people had always bought eggs— a daily necessary in a climate where egg-flip was the usual diet— from a Merchant who came to their gates once a week. And the people always bid wildly against each other: so there was quite a lively auction every time the Merchant came, and the last egg in the basket used to fetch the value of two or three camels, or thereabouts. And eggs got dearer every week. And still they drank their egg-flip, and wondered where all their money went to.

"And there came a day when they put their heads together. And they understood what donkeys they had been.

"And next day, when the Merchant came, only *one* Man went forth. And he said 'Oh, thou of the hook-nose and the goggle-eyes, thou of the measureless beard, how much for that lot of eggs?'

"And the Merchant answered him 'I *could* let thee have that lot at ten thousand piastres the dozen.'

"And the Man chuckled inwardly, and said 'Ten piastres the dozen I offer thee, and no more, oh descendant of a distinguished grandfather!'

"And the Merchant stroked his beard, and said 'Hum! I will await the coming of thy friends.' So he waited. And the Man waited with him. And they waited both together.

"The manuscript breaks off here...". (S&BC, 618-9).

It would be pleasant to record that Carroll's notice of the indeterminacy problem was very fresh. Unfortunately, Schumpeter cites Beccaria as being well aware of the problem, as well as Menger and Bohm-Bawerk and, more analytically, Edgeworth in *Mathematical Psychics* (1881). Moreover, Cournot (1838) includes an account of a similar situation heavily restricted by assumption, in which two monopolists pro-

duce different products bought only as inputs by a third monopolist. (Schumpeter, 1954, 983-4). But while Carroll cannot be credited with a pioneering discovery in this area, it seems likely that his was an independent, though late, discovery.

I have found no evidence in Carroll's writing of any acquaintance with the writing of political economists. While Bartley (1977) showed that Carroll was in correspondence with John Neville Keynes as well as other Cambridge logicians, the material he cited was concerned wholly with logic, not with economics. Carroll owned works by J. N. Keynes, J. S. Mill, William Whewell, and W. S. Jevons on logic, but apparently not their works on political economy (31). Bartley quoted R. B. Braithwaite as saying in 1932 that it was by then the fashion for Cambridge economists and logicians 'to *pretend* to derive their inspiration from' Lewis Carroll (30, emphasis mine). Not only is this considerably after Carroll's death, however, but it is unclear that they were referring to anything but Carroll's logic.

Similarly, Bartley cited Lewis Carroll's acquaintance with Isaac Todhunter only with respect to their agreement on the merits of the Cambridge examination system. Todhunter's 1865 *History of the Mathematical Theory of Probability* was the only work from which Carroll was likely to have learned of Borda's and Condorcet's voting systems (or Waldegrave's minimax solution of a two-person zero-sum game). Kendall (1963) noted that Carroll's copy of Todhunter had only the first few pages cut, so it would seem that Carroll's discussion of schemes identical with theirs was independent.

It might be thought that Carroll could have been in contact with Edgeworth, who was not merely a mathematical thinker but Drummond Professor of Political Economy at All Souls, Oxford. Edgeworth's appointment there, however, began in 1891. This was fairly late for *Sylvie and Bruno Concluded* (1893), especially since Carroll claimed in the introduction to S&BC that most of its manuscript had been in existence since the printing of *Sylvie and Bruno* (1889). Moreover, Edgeworth spent the eighteen years between taking his degree at Oxford and his appointment there away from Oxford. Edgeworth never had a connection with Christ Church (Creedy, 1986). Carroll, an undergraduate at Christ Church and then a Student (fellow) there, led a rather restricted life within Oxford. It therefore seems improbable that Carroll was influenced by any other writer on bilateral monopoly, but he ranks at best as a popularizer.

By isolating some of his remarks on political economy to Outland and Outlanders, Carroll made it clear that he understood the upside-downness they (generally) embodied. Only one argument which has strong preposterous aspects (from an economist's point of view) is not made by an Outlander, and the extent of Carroll's seriousness is difficult to determine here. This argument is made by the Earl, a virtuous character, and therefore, for Carroll, one who can convey valuable insights. The first half of the argument has a sanctimoniousness to it which fits in well with Carroll's more serious essays, but the latter half is clearly intended to amuse. The Earl has

"...yet *another* theory for adding to the enjoyment of Life.... It is, that we should learn to take our pleasures *quickly*, and our pains *slowly*.... By taking *artificial* pain

—which can be as trivial as you please—*slowly*, the result is that, when *real* pain comes, however severe, all you need do is let it go at its *ordinary* pace, and

it's over in a moment!... by taking [pleasure] quick, you can get so much more into life. It takes *you* three hours and a half to hear and enjoy an opera. Suppose I can take it in, and enjoy it, in half-an-hour. Why, I can enjoy *seven* operas, while you are listening to *one!*" (S&B, 512).

The Earl later admits this his enjoyment of music is not independent of its speed—he has heard a broken music box pour out its piece in three seconds, but did not enjoy it, not having been 'trained to that kind of music!'

What is of economic interest here is the consideration of time as a factor in consumption. The 'slow pains' principle suggests that Carroll felt that the duration of an experience is one of its characteristics, influencing the utility it provides. The 'fast pleasures' principle suggests that Carroll thought of consumption as independent of the time it takes, but this is exploded by the music-box example. Carroll's view of time as a characteristic of consumption is consistent with his explication of the characteristics analysis of services, which is discussed in the second section following.

Carroll's belief that taking on voluntary disutility now ('artificial pain') is worthwhile so that later disutility will not seem so unpleasant seems odd from the ordinary economist's point of view. Surely 'taking the artificial pain slowly' must mean prolonging an unnecessary discomfort. It seems strange that, even if we can accept this procedure as likely to render the consumer less subject to disutility from pain, it should have so much value with a non-negative rate of time preference. (Presumably Carroll would argue that by prolonging *trivial* pains, the cost is low but the desensitizing effective, like a sort of inoculation.) Carroll's outlook on pain is very much in line with the Christian perspective which informs his more serious texts on political economy: for Christians, temporal pain yields eternal bliss.

The Dicta of Arthur

As mentioned above, Carroll claimed that 'most of the substance of *both* Volumes [of *Sylvie and Bruno*] was then in existence in manuscript: and [his] original intention was to publish the *whole* story at once' (S&BC, 538). An economist-reader who notes structural change between volumes may find this dubious, however: in S&BC, Arthur, Carroll's hero, spouts a number of seriously intended diatribes on political economy, while in S&B he contents himself with criticizing High Church service and making word plays and other jokes. There seem to be two possible reasons for Arthur's change of conversational matter. One is that Carroll's feeling that he ought to and could preach to the general public increased after the publication of S&B, and that he revised S&BC accordingly. The other is purely internal. Carroll, after having his heroine reject her erstwhile follower for Arthur and Arthur's superior virtue, felt that Arthur should be shown to be very serious, as well as an entertaining man. In this case, Carroll need not have substantially altered his draft, but then the gap between publication of the two books becomes puzzling.⁵ I have no idea which hypothesis should be favoured: they seem equally likely.

One of Arthur's longest dissertations is on the subject of whether people who have inherited enough wealth to support them have a moral right not to work. Arthur feels that it is ethically better for them to work, but that they have every right not to.

"The whole subject... of what we may call 'idle mouths' (I mean persons who absorb some of the material *wealth* of a community— in the form of food, clothes, and so on— without contributing its equivalent in the form of productive labour) is a complicated one, no doubt. I've tried to think it out. And it seemed to me that the simplest form of the problem, to start with, is a community without *money*, who buy and sell by *barter* only; and it makes it yet simpler to suppose the food and other things to be capable of *keeping* for many years without spoiling.... The commonest type of 'idle mouths'... is no doubt due to money being left by parents to their own children. So I imagined a man... who had contributed so much valuable labour to the needs of the community that its equivalent, in clothes, etc., was (say) five times as much as he needed for himself. We cannot deny his *absolute* right to give the superfluous wealth as he chooses. So if he leaves *four* children behind him..., with enough of all the necessaries of life to last them a life-time, I cannot see that the *community* is in any way wronged if they choose to do nothing in life but to 'eat, drink, and be merry'. Most certainly, the community could not fairly say, in reference to *them*, 'if a man will not work, neither let him eat.' Their reply would be crushing. The labour has already been *done*, which is a fair equivalent for the food we are eating; and you have had the benefit of it. On what principle of justice can you demand *two* quotas of work for *one* quota of food?"

"I suppose the *second* form of the problem is where the 'idle mouths' possess *money* instead of *material wealth*?"

"Yes... and I think the simplest case is that of *paper-money*. *Gold* is itself a form of material wealth; but a bank-note is merely a *promise* to hand over so much *material wealth*... [Suppose the active father above earned five thousand pounds, spent one thousand, and left four thousand to his four children.] ...surely they have a full right to present these written promises, and to say 'hand over the food, for which the equivalent labour has already been done'. ...I should like to drive it into the heads of those Socialists who are priming our ignorant paupers.... I should like to *force* them to see that the *money* which "those 'aristocrats'" are spending, represents so much labour already done for the community, and whose equivalent, in material wealth, is *due from the community*." (S&BC, 559-560).

Carroll's analysis clearly bears comparison with that of Locke. They have a somewhat similar sound, but Locke contrasted the incentive effects on the 'father' of the legal right to bequeath with its absence, while Carroll was concerned exclusively with the children's right to consume his surplus production. The crux of Carroll's argument was that the 'idle mouths' contribute nothing to their society, but that they take away from it nothing that they have not been given by another contributor. This suggests that Carroll would disagree with Samuel Johnson and Adam Smith's *The Theory of Moral Sentiments* on the benefits to society and spur to economic growth supplied by the consumption of the wealthy, and this is indeed the case. Later, in his diatribe on charity, Arthur says that this idea is based on fallacies 'which have gone so long uncontradicted that Society now accepts [the idea] as an axiom!' (S&BC, 561).

Carroll's analysis embodies a false dichotomy and a particularly strong implicit assumption. He began by separating the cases of a non-monetary and a monetary economy, but the distinction makes no difference to his reasoning. (Like many economists and virtually all laymen of his period, Carroll believed in an excessively

strong distinction between fiat money and gold, and this seems to have motivated his dichotomy.) Carroll, a staunch Conservative, made this distinction the basis of his reproach to 'Socialists'. He has made an implicit assumption, however, which renders any comparison of his system with that of Marx or lesser radicals meaningless. The only goods in Carroll's hypothesized economies are 'food, clothes, and so on'—that is, consumption goods. He made no mention of capital at all, either as a factor in production (which is conveniently vague) or as a potential purchase of the father. Nor could Carroll's analysis handle such a complication. If the heirs received capital, hired labour to use it to produce, and sold the output, they could remain comparatively idle while gaining income. Does their father's surplus production entitle them to have their own labour multiplied in this fashion? Carroll's discussion cannot tell us. A greater problem is that, if the father chooses to purchase capital with a portion of his surplus production rather than pure consumption goods, the benefits his heirs gain from the capital result not from the father's surplus per se, but from his choice of purchases. Carroll might well have trouble justifying the morality of this proceeding within the framework he erected. In addition, assuming away capital permitted Carroll to escape entirely from the problems of exploitation which the 'Socialists' addressed.

Carroll's Christian political economist Arthur has a great deal to say about the moral credit people deserve for different types of charity (S&BC, 561-563). He articulates the belief that giving is not charity unless the giver forgoes something: that is, that the effect on the donor is more important than the effect on the recipient in deciding whether a gift is charity. Carroll, a frugal man himself, felt that satiation was fairly common. If, however, an otherwise satiated donor denied himself the pleasures of that 'morbid craving', hoarding, then he could be said to do good by his gift. Carroll's whole outlook on the morality of charity depended on the intentions of the giver. In examining Charity Bazaars, he had Arthur note that those who produced goods and acted as salespeople for some cause acted charitably. Those who purchased the goods, however, were charitable only to the extent that the prices they paid exceeded the market prices of the goods they bought. Arthur angrily slashes some weeds with his walking stick, saying,

"And my opinion is that it is not given in the best way: far better buy what they choose to *buy* and give what they choose to *give*, as two *separate* transactions: then there is *some* chance that their motive in giving may be real charity, instead of a mixed motive—half charity, half self-pleasing." (S&BC, 563).

Similarly, Arthur's theory of culpability for crime is one in which those who have less temptation and less chance of being found out for committing some wrong are more culpable for doing so than those with more pressing need and a greater chance of being detected. Arthur explains that this follows from the principle that '...a man is responsible for his act of choosing, but *not* responsible for his environment'. (S&BC, 590-1). The degree of a person's responsibility for evil acts depends on her efforts to resist temptation.

Carroll's view of moral accounting as depending more on the attitude of the actor than on the nature of her acts was a common one among those English clergy who were not evangelical Non-Conformists or Low Church. Catholic confession is based on the concept that the effect not merely of acts but of thoughts on their perpetrator is the important moral issue. George Macdonald, a prominent, though not High

Church, preacher wrote in his religious novel *Lilith* that it was as bad of a boy to want to eat the 'wrong kind' of apple as to do it, so that restraining him would not be helpful. This same viewpoint survived in the *Principia Ethica* of G. E. Moore.

Like many nineteenth century political economists, Carroll was concerned with the problem of drink for the lower classes. Unlike many, Carroll did not advocate teetotalism for workers. He was Curator of the Christ Church common room from 8 December 1882 to the end of 1892, and managed the cellar with great enthusiasm and care for dons's tastes and wallets. He did not feel that teetotalism was necessary or desirable for anyone. Carroll's concern about the drink problem was for the worker's excessive expenditures and possible drunkenness, both of which he thought could be avoided without total abstinence. Carroll's narrator tells a tutelary story:

... a certain cottager... bought himself a little barrel of beer, and installed his wife as bar-keeper: and... every time he wanted his mug of beer, he regularly paid her over the counter for it: ... she never would let him go on "tick", and was a perfectly inflexible bar-keeper in never letting him have more than his proper allowance: ...every time the barrel needed refilling, she had plenty to do it with, and something over for her money-box: ...at the end of the year, he not only found himself in first-rate health and spirits, ...but had quite a box full of money, all saved out of his own pence! (S&BC, 571-2).

It was probably because his own favourite company was that of girl children that Carroll overlooked the social aspect of going to the pub. He seems to have felt that the social interaction which occurred was merely peer pressure and debauchery.

Carroll exhibited both a basic understanding of the national income identity and his dislike of teetotalism in the following exchange.

[Teetotaler:] "Read this card.... The stripes of different colours represent the amounts spent on different varieties of food. Look at the highest three. Money spent on butter and on cheese, thirty-five millions: on bread, seventy millions: on *intoxicating liquors*, one hundred and thirty-six millions!... That's where all the money goes to!"

[Arthur:] "Have you seen the *Anti-Teetotal Card*?... [It is] almost exactly like this one. The coloured stripes are the same. Only, instead of the words 'Money spent on', it has 'Income derived from the sale of'; and instead of 'That's where all the money goes to', its motto is '*That's where all the money comes from!*'" (S&BC, 597).

In his most risqué facetious sally, Arthur proposes a new institution which would permit bourgeois and upper-class people to gain more information on each other prior to making a decision on marriage than current conditions permitted.

"Let X be the gentleman, and Y the lady to whom he thinks of proposing. He applies for an Experimental Honeymoon. It is granted. Forthwith the young couple—accompanied by the great-aunt of Y, to act as chaperone—start for a month's tour, during which they have many a moonlight-walk, and many a *tête-à-tête* conversation, and each can form a more perfect estimate of the other's character, in four *weeks*, than would have been possible in as many *years*, when meeting under the ordinary restrictions of Society. And it is only after their *return* that X finally decides whether he will, or will not, put the momentous question to Y!... [Where no marriage occurs,] an unsuitable match would be

prevented, and both parties saved from misery!" (S&BC, 596).

Arthur suggests no similar plan for the lower classes; nor, according to his rationale, would it be so needed for informational purposes. If a critic of Arthur's is correct in thinking that the trial honeymoon would forestall nine out of ten marriages, then this plan would tend to delay marriage for the middle and upper classes, but would not affect the majority. This makes Arthur's scheme a curious counterpoint to Malthus's and Mill's concern for the delay of marriage among workers.

The propositions in political economy which Carroll put in Arthur's mouth are sometimes interesting, almost always didactic, and typically flawed by Carroll's excessive reliance on logic and 'common sense'. There is no indication that Carroll ever read Ricardo, either Mill on economics, other nineteenth century economists, or even Smith. Like many laymen, Carroll assumed that, by the light of his Christianity and logic, he could inform the public. Much of the material which combines the two is interesting only from a Christian point of view.

Characteristics Analysis of Services

From 1880 to 1885, Carroll published logical and algebraic problems in the magazine *The Monthly Packet*. These puzzles (in installments called 'Knots') were structured around three sets of characters who converged in the last Knot: the Elder and Younger Travellers, Clara and her aunt Mad Mathesis, and the boys Lambert and Hugh with their tutor, Balbus. In Knot Six, the two travellers are obliged to rate Kgovjnan weavers of scarves according to three characteristics of their services:

"The yearly competition for the post of Imperial Scarf-maker is just ended; you are the judges. You will take account of the rate of work, the lightness of the scarves, and their warmth. Usually the competitors differ in one point only. Thus, last year, Fifi and Gogo made the same number of scarves in the trial-week, and they were equally light; but Fifi's were twice as warm as Gogo's and she was pronounced twice as good. But this year, woe is me, who can judge it? Three competitors are here, and they vary in all points!... Lolo makes 5 scarves while Mimi makes 2; but Zuzu makes 4 while Lolo makes 3! Again, so fairy-like is Zuzu's handiwork, 5 of her scarves weigh no more than one of Lolo's; yet Mimi's is lighter still— 5 of hers will but balance 3 of Zuzu's! And for warmth one of Mimi's is equal to 4 of Zuzu's; yet one of Lolo's is as warm as 3 of Mimi's!" ('A Tangled Tale', Knot Six, 328).

Clearly, the services of the weavers are being rated rather than the scarves they make: otherwise the speed with which they work would not figure in the analysis. (This distinguishes the scarf problem from the Earl's discussion on the duration of pleasures and pains. The speed with which a scarf was made could not influence one's utility from use of the scarf.) The Kgovjnians had no difficulty in choosing the best weaver in the first instance, which corresponds to the absolute advantage case of gains from trade, but were stumped by the second case of Ricardian comparative advantage.

According to the canonical economics of the period, Lolo's, Mimi's and Zuzu's scarf-making should have been considered different services, and not strictly comparable. To Carroll, however, their services were bundles of three attributes which were comparable. Almost by accident, Carroll anticipated Lancaster's (1966) intro-

duction of characteristics analysis. Von Thunen's analysis of the returns on grain differing with the distance of the producer from the market is the closest earlier analogue, and was surely unknown to Carroll.

Carroll's passage on how 'Fifi's [scarves] were twice as warm as Gogo's and she was pronounced twice as good' indicates only that $U(Sg, Lg, 2Wg) = 2U(Sg, Lg, Wg)$, where S is speed, L lightness, and W warmth. $U = W(S+L)$ satisfies this condition. If we assume that doubling any one of the characteristics results in a doubled rating, we have a somewhat more restrictive condition. Yet even this is insufficient to allow us to compare the bundles. To Carroll, however, the method of solving the problem was unique and almost self-evident:

L makes 5 scarves, while M makes 2; Z makes 4, while L makes 3. Five scarves of Z's weigh one of L's: 5 of M's weigh 3 of Z's. One of M's is as warm as 4 of Z's and one of L's as warm as 3 of M's. Which is best, giving equal weight in the result of rapidity of work, lightness, and warmth?

Twenty-nine answers have been received, of which five are right, and twenty-four wrong. These hapless ones have all (with three exceptions) fallen into the error of *adding* the proportional numbers together, for each candidate, instead of *multiplying*. Why the latter is right, rather than the former, is fully proved in textbooks, so I will not occupy space by stating it here: but it can be *illustrated* very easily by the case of length, breadth, and depth. ('Answer to Knot Six', 356).

Note that Carroll seems unaware of having added a new assumption in the answer to the Knot: 'equal weight' for the characteristics. Apart from this, Carroll was certainly correct in feeling that the remark about double warmth rendering Fifi twice as good a scarfmaker rules out simple addition of the proportional numbers. He was wrong, however, in thinking that textbooks 'prove' that preferences must be of the form he assumed, $U = aSLW$, where a is any positive constant. Indifference curves were described by Pareto in the 1890s, but had not yet appeared in the economics literature at this point. Had Carroll thought in terms of level curves for ratings of weavers by characteristic, he might have been less likely to make such a strong supposition about preferences implicitly. Despite this handicap, Carroll's idea of characteristics analysis was an astonishing one to be presented at the time. Even had the outlet for this framework been a serious one, it seems unlikely that this type of analysis could have caught on at the time.

Lawn Tennis and Voting

Black (1958) briefly mentioned a pamphlet by Carroll on a more equitable plan for lawn tennis tournaments (Carroll, 1883) as merely illustrating Carroll's work on voting theory, and Black (1970, 4) says that 'the mode of thinking is the same as in the theory of committees (1873-76), the two problems being isomorphic'. The two problems are not isomorphic, and this is a misrepresentation, although some of the reasoning style in (Carroll, 1883) is similar to that of the early pamphlets. Carroll was well aware that voting was a process for arriving at a consensus choice for a group of people who had differing, but valid, ratings of the candidates. He quite rightly viewed a tennis tournament as a testing procedure for ranking at least the top four candidates with respect to their concrete abilities. Since what is tested in a tennis

tournament is a quality which lies beyond the choice process, tennis tournaments are not much like voting. (In fact, the problem has some analogy to the problem of reducing the number of operations in computer sorting.) While some concerns, such as the effect of the order of pairwise comparisons, are relevant for both tennis and voting, Carroll's solutions to the two problems are quite different, and he seems (correctly) to be more satisfied of the solidity of his proposed method in the case of tennis.

Throughout most of this pamphlet, Carroll assumed that the players's performance was not stochastic, that is, that the player ranked 2 would always beat players ranked 3 through 32, but be beaten by the player ranked 1. This assumption acts as a *reductio*: even where there is no doubt that a particular player 'deserve' second prize, the testing procedure of the tournament may return an incorrect answer. Non-stochastic play also meant that Carroll's implicit assumption of transitivity was perfectly well founded. Since intransitivity is an important and scarcely eliminable problem with voting systems, nonstochastic play makes a vast difference to the concerns of a tennis tournament.

Carroll began with a proof that, under the usual tournament rules, while the best player would always win, the second, third, and fourth best players would by no means necessarily win a prize. The usual tournament rules for a 32 player event, like those today, involved sixteen matches in the first round. Losers were eliminated in each round. The second round had eight matches, the third four. All four players in the fourth round were sure of a prize, and the fifth round determined which of the two fourth round winners came in first and which second, while the fourth round losers were matched to determine third and fourth prizes.

Carroll proved by construction that, if we number players 1 to 32 for their rank, start by matching 1 with 2, 3 with 4, and so on, and proceed the same way in each match, 1 will win first prize, 17 second, 9 third and 25 fourth. More generally, he argued that

...if we divide the list of competitors, arranged in the order in which they are paired, into 4 sections, we may see that all this method really does is to ascertain who is best in each section, then who is best in each half of the list, and then who is best of all. ('Lawn Tennis Tournaments' (LTT), 1060).

If we assume that pairings are done at random, the probability that player 2 will receive second prize (that is, not meet 1 until the last round) is only $16/31$.⁴ If, Carroll says, it is argued that the extra stochastic element here is unobjectionable, let the second best shot in a rifle contest be obliged to draw from an urn to determine whether she will get her prize, and see if she objects. (In a letter of 15 July 1887 to an unknown correspondent, Carroll stated that one or two letters to the *St. James Gazette* by the well-known authority 'Cavendish' after his tennis pamphlet came out indicated that ordinary method, 'though known to substitute "chance" for "skill" to a great extent, is deliberately preferred by most players. That fact seems to me to make the matter unworthy of any further investigation.' (*Letters*, 682)).

Carroll's method for gaining a proper order for the four prize winners is of course a procedure of pairwise comparison, but he suggests giving only three prizes, and the system for dropping players from the tournament is quite different.

A list is kept, and against each name is entered, at the end of each contest, the name of any one who has been superior to him— whether by actually beating

him, or by beating someone who has done so.... So soon as any name has 3 "superiors" entered against it, it is struck off the list. (LTT, 1061).

In subsequent rounds the unbeaten are paired together, then those with one superior, then those with two, 'avoiding, as far as possible, pairing two Players who have a common superior' (LTT, 1061). Thus in the first round, the set of players is partitioned into those who may, and those who may not, be the best player. After the second round, those who may be 1 are half as many, those who may be 2 are of the same number as before, and as many may be 3 as may be 1. Subsequent play will refine these sets further.

Carroll's idea in determining elimination was that those players with three superiors before the final round cannot be ranked 3 or above. As a general rule, players should be eliminated only when they have as many superiors as there are prizes. Thus, Carroll's scheme is invariant to the order of pairing under the assumption of non-stochastic play.

Carroll also objected to the conventional game-set-match scoring⁵ (similar to that still used). He felt that it was 'liable to lead to unjust results' because a player could win the majority of games and still lose the match, and showed this by construction. Carroll's concern here is very sensitive to what is meant by 'best player'. Under the game-set-match convention, a player who can win more matches than her opponent is superior. If, as Carroll suggested, a player wins a contest when she has been first to win a particular number of games or gets so many games ahead, then she is adjudged superior for having won more games than her opponent. It would also be possible to decide that a player should win a contest based on *points* won, in tiebreaker fashion. The criterion of choice depends on how we rate timing and psychological factors in determining a player's quality.

Carroll's concern with game-set-match scoring does have a political analogue. The Parliamentary system of electing a Prime Minister indirectly, as leader of the party which has won enough seats to form a government, is similar.⁶ The American system of electoral colleges which intervene between voters and election results geographically is even more similar to game-set-match scoring. Carroll did not address these problems in any of his work. If he were to argue by analogy to his scoring plan, he would presumably suggest direct election of the U.S. President, and either separate direct election or the counting of party votes across districts to determine the Prime Minister. In either case, a parliamentary system would lose the advantage of having a national leader with guaranteed adequate support in the house, but there might be advantages to balance the loss.

While Duncan Black thought the tennis pamphlet a mere extension of Carroll's voting work, it addressed a very different problem: how can we rank the top m of n candidates in terms of an attribute measurable in principle by pairwise comparison? This question is of even more interest if we consider the possibility that the results of each pairwise comparison are stochastic. In the case of tennis, this would mean that players might perform better or worse than their precise rating, on the basis of chance. In the case of comparing the weights of rocks, stochastic pairwise comparison could result from measurement error. Carroll addressed this sort of issue only tangentially. He suggested that the player ranked 5 would feel her chances better if playing above her rating would guarantee her a prize, than if pure chance might give her a prize as high as second (LTT, 1066). Certainly, Carroll's method would be less likely than the conventional one to produce gross errors in this case.

Conclusion

Carroll's pamphlets from the 1870s on voting are well known and *Principles of Parliamentary Representation* (1884) has been discussed by Duncan Black. While Carroll's skills as a logician served him well in this field, his writing on concerns aside from public choice within political economy is of interest. The paragraphs scattered through *Sylvie and Bruno* and *Sylvie and Bruno Concluded* are amusing or worth thinking about, even when not particularly useful. In *A Tangled Tale*, Carroll gave the first discussion of the characteristics analysis of services. His pamphlet on lawn tennis, while using reasoning common to the 1870s pamphlets, approached the problem of ranking things according to a theoretically measurable characteristic by pairwise comparison.

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Notes

1. I follow Black in referring to Dodgson as Carroll, regardless of how his works were signed. Those works which I know to have been signed as by Dodgson are so indicated in the references. It should be noted that several such works (as well as early poems and stories published under different pseudonyms) are reprinted without the signature being noted in *The Works*. In following this policy I burke the issue of "dual personality" (Dodgson versus Carroll) in my subject's work.
2. In some English colleges, such as All Souls, the head is called the Warden.
3. It may be accounted for by Carroll's habit of fretting his illustrators. Harry Furniss, who illustrated the *Sylvie* books, said that Carroll was the most troublesome of his authors by far.
4. In modern tennis tournaments, the seed system, though imperfect, is used to reduce the chance of inappropriate prizes in this sense.
5. Whereby a set was the best of 11 games and a match was the best of 5 sets.
6. I am deliberately ignoring the complication that the leader must have won a seat, herself. In practice this has not been important.

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