The Process of Collaboration Between Oskar Morgenstern and John von Neumann

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1. Introduction

This article investigates the process of collaboration towards the birth of the book, *The Theory of Games and Economic Behavior* (TGEB), written by J. von Neumann and O. Morgenstern in 1944, with particular emphasis on the development of the 'game' concept. Three stages are distinguished in the collaboration: the stage of games of chance, of games of strategy, and of games of bargaining with their solution concept. According to my working hypothesis, the ideas of O. Morgenstern were important at each stage for the development of the game concept, while the general framework had already been structured by the article of J. von Neumann in 1928.

The second stage was perhaps the most important for their collaboration. The two met with their separate backgrounds, which were rather different but nevertheless in part similar to each other. In the third stage, though the contribution of O. Morgenstern can still be appreciated, for example in the solution concept of stable sets, his research interests were largely dominated by those of J. von Neumann. It seems to be a typical destiny of collaboration that it tends to domination by one partner. Here, because of the limits of space, I would like to focus on the second part of the collaboration, while occasionally reflecting on the first part as a critical factor in its pre-history.[1]

2. On chance: the Austrian Tradition

(a) Pre-history: D. Bernoulli

Regarding the first stage of games of chance, there were several early contributors to this subject such as B. Pascal, P. de Fermat, C. Huygens, who collectively defined the concept of "equivalent gambles", but we will not go into detail here.[2] What attracts our attention is the approach of D. Bernoulli expressed in his article published in 1738, about the measurement of risk, resulting from some correspondence in letters on mathematical issues.[3] *TGEB* mentions the contribution of D. Bernoulli (p. 83) and we too must concur that he deserves acknowledgement as the first precursor in this connection.

The article by Bernoulli also became well-known among economic theorists as well from the middle of the nineteenth century on, because it developed a concept similar to that of marginal utility.[4] It dealt with the problem of why people behave in deviation from mathematical expectation. D. Bernoulli insisted that what mattered in a risky situation was not the money one could receive but the utility one could derive from it. He stated further that this utility function had to be concave because its rate of increase would decrease as the quantity became larger, which he called the "moral expectation" (Bernoulli 1738, 1956, p. 25).

The point Bernoulli made is that the consideration of marginal utility can be applied to risky situations as a kind of game yielding a profit. It is a very interesting point; he had previously raised it in connection with a different problem concerning marine insurance (M. Suzuki 1994 p. 397). In his discussion of the game there was a player who tossed a coin

repeatedly and another player who could win some money depending on the outcome, and the value that the latter should lay on this game in terms of certainty equivalence was brought up. This game at first sight looks like a two-person game, but in fact it is only a game of chance where only the second player has real significance. The first player, who only tosses the coin, could also be called Nature, because the outcome depends purely on chance and not on the strategy of a conscious and intentional actor. Bernoulli missed the point that there had to be a symmetry of treatment of each player for his situation to be treated as a game of strategy. We will discuss this matter below.

This way of thinking of a game of chance in terms of the equivalence of a risky event with a sure one in a decision-making process was rather new and important for economic theorists in the middle of the nineteenth century. Only gradually were they becoming aware of its importance for the theory of marginal utility in general.[5]

It was only in 1927, when K.Menger made a report on the subject of the St. Petersburg's Paradox of D. Bernoulli's game to the Viennese Economic Society, that the problem was put in order. He introduced a psychological element into the discussion by emphasizing the tendency of people to gamble, together with the comparison of a certain amount of money with a choice of a gamble or a lottery. This was a big step preparing the way for introducing games of chance into the field of economics, and it was later developed systematically in *TGEB*.[6]

(b) Böhm-Bawerk on Lotteries

Here we have to make one proviso immediately: though the problem of games of chance was not so well-known in general among economic theorists, we can find an exception in an early work of Böhm-Bawerk. He had already conceived of the idea of certainty equivalence and Morgenstern took over the notion from him. Böhm-Bawerk seemed to be neither fully conscious of the importance of games of chance nor did he develop the idea itself comprehensively, but he did give an example of a lottery that has some importance for our investigation. We can call him the second precursor of the authors of game theory. He mentioned this idea in a small book in 1881 on rights and relationships from the viewpoint of economics, i.e. a book about property rights, wealth and the valuation of assets.[7] There property was defined as the sum of goods and wealth, which were provided for some future need. The problem to be solved was to measure the value of a good which had neither come into existence yet nor could be used at that moment, but would be at hand in the future. In other words, Böhm-Bawerk approached the economic problem of valuing a future good from a legal point of view.

This connection with jurisprudence was normal at the time for economic theorists (Streißler 1969 p. 257) in Austria, and in his value theory, too, he followed the Austrian tradition. He developed a subjective value theory, which insisted that the value of a good was based on the utility placed on it, together with several conditions that this good had to exist, had to be already at hand, and so on.[8] In this sense, Böhm-Bawerk's contribution to games of chance can be seen as a natural development in the Austrian tradition.

Böhm-Bawerk claimed that the classification and valuation of goods depended on how certainly a good was to be delivered into one's hand, or by the quantity of the expected utility attributed to the good. At this point he referred to the idea of compensation for some improbable but more profitable utility, compared with another more probable but less profitable one. It was this comparison that led him to develop an example of the compensation necessary for a real estate parcel that would be gained shortly, that is, fairly certainly, on the one hand, compared with a lottery ticket that would objectively give only a probability (expectation) of profit on the other hand.

It might just be coincidential that Böhm-Bawerk used the example of a lottery, because he did not go any further in developing the concept of a game. But he was surely conscious of the fact that the lottery allowed a chance of gain and it was, as we know today, the very motivation for taking part in such a game of chance.[9] But what Böhm-Bawerk missed in this book was the point that the element of time was essential for considering compensation, a fact which he himself recognized in another context, and, indeed, for which he would become famous, namely in connection with his criticism of the Ricardian-Marxian labor theory of value. Actually, the concept of time, especially in the sense of a proper division of time periods, is very important for the game concept.

(c) Morgenstern on the Element of Time

Within this stage in preparation for the theory of games of chance, according to my working hypothesis, Morgenstern had given much more attention to time than the other author of *TGEB*, John von Neumann. From the viewpoint of the development for *TGEB*, it is one thing to recognize that the problems of games of chance reduce to mathematical computations, but this is not sufficient in itself to establish game theory in full. The need to go beyond mathematics might have been the reason why J. von Neumann did not pay much attention to time. The second preparatory stage for developing the theory of games of strategy is far more important for both authors. But I nevertheless would like to bring the contribution of O. Morgenstern into focus, because it highlights an important aspect of the background of *TGEB*, viz. his consideration of the utility concept.

With historical evidence, it is fairly certain that the two theories of D. Bernoulli introduced by K. Menger on the one hand and that of Böhm-Bawerk on the other, were direct sources used by Morgenstern in developing the idea of games of chance. Like these precursors, Morgenstern also seems not to have been conscious of the fact that the phenomena he treated could be defined generally as a game, but rather directed himself to improve subjective value theory, following the traditional approach of the Austrian School of Economics.[10]

Morgenstern wrote an article on the subject of time in 1934, which was an attempt to take risk, uncertainty, mistakes, psychological factors such as temperament, caprice, daring and so on, and expectations of the future into consideration. He had been of the opinion since investigating economic forecasting (Morgenstern 1928), that people did not possess perfect foresight. But he still tried to find some rational criterion, maybe at best the minimum of which one could be assured, similar to the assumption of bounded rationality in our contemporary usage of the term.[11]

The contribution of O. Morgenstern here consists in putting the idea of compensation of E. von Böhm-Bawerk into the appropriate context of the division of time for the budget of an individual, different from the legal situation when faced with the need to evaluate an asset. In contrast to the orderly line of precedence in legal asset valuation, he maintained the 'congruent' (Morgenstern 1934; 1976a p. 156) division of time, using an example of some future scarcity of greater value than the whole income of one period, which necessitated some budgeting over several periods. Like the treatment of contingent goods, as we know them in current research, he insisted that one had to attach a certain time-index to the realization of future needs satisfaction.

Further, this idea of individual planning could also be extended to the process of production because it was defined, in the Austrian connection, as a gradual transformation of goods. Morgenstern came to point out in this connection that the expectation of other traders had also to be considered at the same time, thus criticizing Böhm-Bawerk.[12] According to him, because there necessarily had to be some interaction with competitors and suppliers to estimate factor inputs in the process of production, a comparison with their compensation

rates became necessary. We can see here that O. Morgenstern was already approaching the notion of games of strategy. We conclude that up to this point that this first stage of the development of the theory of games of chance was an important preliminary to the collaboration between the two authors of *TGEB*.

3. On Games of Strategy: the Main Part of the Collaboration

As we have already suggested, our reconstruction emphasizes that the second stage of games of strategy was the most important part of the collaboration, in which different but also strongly congruent interests met and gradually merged together into a common one. The decisive difference between games of strategy and those of chance is that the fate of a player depends not only on this own decisions and actions, but also on those of the other(s), whose behavior is motivated by the same selfish interest as that of the former. Each player must deliberate on his own decisions and actions, or his strategies, taking this fact into consideration. In this sense, the simple unit of analysis becomes the 'strategic interdependence' (Dimand and Dimand 1996, p. 2) between two symmetrical players. The potential collaboration begins, according to my working hypothesis, with the article written by J. von Neumann in 1928, while O. Morgenstern in the same year also began to think of problems similar to those of von Neumann.

(a) J. von Neumann on the Mini-max Theorem

In the article in 1928, J. von Neumann established a general framework for games to be analyzed. He assumed two players, one of whom took the max-min strategy that assured him not more than the value *V1*, while the other took the mini-max strategy that assured him not less than the value *V2* (in the zero-sum, two-person game). He proved the mini-max theorem that there was a saddle point *V1= V2* in general which he proposed as the value of a game, as the outcome of forces trying to tug from two opposite sides (J. von Neumann 1928, 1959 p. 21). This was about two hundred years after a similar theme had been analyzed in the form of a then-current game called 'le Her'.[13] The general proof of the mini-max theorem was composed for the first time by J. von Neumann for this article, though there were some weaker contemporaneous contributions.[14]

J. von Neumann found the 'strategies' of the players confronting conflicts of interest as the essential aspect of games and intended to widen the range of phenomena that could be analyzed as games. We have evidence to show that he was conscious of economic applications, for example in his usage of the word "homo economicus".[15] This strongly corresponds to the concerns of Morgenstern. The aspect of a conflict of interest between players in a game of strategy became still clearer when von Neumann studied a game with three players. There he constructed an artificial two-person situation, i. e., one person against a couple consisting of the other two players, and he applied the framework of the mini-max theorem. In addition the possibility of forming coalitions was also introduced here. J. von Neumann stated that any coalition must be continuously exposed to the danger of being replaced by another, because the formation of the latter could be as beneficial as the former for both members of this coalition. These considerations correspond to those of Morgenstern fairly well.

(b) Morgenstern's Scepticism of Timeless Equilibrium Analysis

As already indicated, Morgenstern came to analyze a similar problem through his investigation of competition and the time structure of economic theory. In 1935 he wrote an important, long-neglected article on the relationship between the assumption of perfect foresight and equilibrium. There he compared different equilibrium theories with and without

this assumption, and came to the conclusion that this assumption was neither necessary nor sufficient for the construction of equilibrium theory (Morgenstern 1976b p. 174).[16] It was this scepticism about the assumption of perfect foresight and about timeless adjustment or "reaction velocity of infinite magnitude" (Walras) towards an equilibrium point that moved Morgenstern to study the conditions and adjustment processes in detail, and he was subsequently to become aware of the importance of mutual reaction and interaction among market participants. He came to the conclusion that this reaction and interaction did not necessarily lead to a definite unique equilibrium point. By the way, in connection with the third stage of collaboration, it was precisely consciousness of a similar problem that was shared by the authors of *TGEB*.

Morgenstern tried to revive the normal and correct sense of the word 'competition'. He recognized that the so-called Walras-Pareto concept was unsatisfactory, because it assumes that "everyone faces fixed conditions, given prices, and has only to adapt himself to them so as to attain an individual maximum which may even be zero as in the case of profit" (Morgenstern 1976e, p. 275). This led him to investigate the eventual adjustment process to an equilibrium which embodied a slower reaction velocity. The mutual reaction of the participants was very important here, because competition normally meant "a struggle with others, fight, rivalry, attempting to get ahead, or at least to hold one's place" (ibid, p. 268).

Like J. von Neumann, Morgenstern stressed that players could not completely control outcomes; that there was nevertheless some room for players to influence outcomes by their own decisions because of the interdependency of their fates. So there was an open problem here for each player. How much should each player think of the potential actions of an opponent, when he and his opponents had to make the decision at the same time? If there were two players, say, A and B, A had to simulate the thinking processes of B, while B also had to simulate the processes of A, which possibly included A's simulation, and so on. This naturally creates an infinite regression. Morgenstern demonstrated this simulation process with his favorite example of a famous passage of A. C. Doyle's concerning Sherlock Holmes and Moriarity; he used this three times, first in the book in 1928, then in the article in 1935 and again in *TGEB* in 1944.

> "Sherlock Holmes, pursued by his opponent, Moriarity, leaves London for Dover. The train stops at a station on the way, and he alights there rather than travelling on to Dover. He has seen Moriarity at the railway station, recognizes that he is very clever and expects that Moriarity will take a faster special train in order to catch him in Dover. Holmes' anticipation turns out to be correct. But what if Moriarity had been still more clever, had stimulated Holmes's mental abilities better and had foreseen his actions accordingly? Then, obviously, he would have traveled to the intermediate station. Holmes, again would have had to calculate that, and he himself would have decided to go on to Dover. Moriarity would again have 'reacted' differently" (Morgenstern 1928 p. 98, 1976b p. 173-174, *TGEB* p. 177).

Morgenstern understood this problem as an informational one, by considering and ranking the thought experiments of the players. This is different from the idea of merely requiring the assumption of perfect rationality and setting up a ranking by an infinite regression. To be sure, if we represent the problem according to the framework of the mini-max theorem, then we would arrive at the saddle point as the outcome (*TGEB* p. 177) and we should regard this as the real starting point of the collaboration.[17] But the 'solution' provided in *TGEB* for the Holmes-Moriarity problem did not fully satisfy Morgenstern because it was too static and deterministic (Rellstab 1992b p. 53). The two authors of *TGEB* confronted a similar problem and developed a common 'solution' for it, which suited von Neumann more than it did Morgenstern and which Morgenstern accepted only for the time being.[18]

Morgenstern's dissatisfaction with the *TGEB* 'solution' notwithstanding, his early insights contributed significantly to the concept of a 'game' in today's usage. Morgenstern emphasized that the thought experiments among the players need to be analyzable without infinite regression in order to be applicable to humans. This seems to correspond, as we have already seen, to attributing a kind of bounded rationality to individuals.[19] As we now know, one of the current tendencies of game theoretical literature, the evolutionary theory or dynamic programming approach based on bounded rationality, takes the complex learning process into consideration, and increasingly celebrates the early contribution of Morgenstern.[20]

(c) A Remark on the Diffusion of Knowledge in a Society

As evidence that Morgenstern was fully conscious of the informational problems which have much to do with the current idea of evolutionary mechanisms based on bounded or reduced rationality, it is pertinent to comment on his ideas on the diffusion of knowledge in a society. He called the problem "theory absorption" and considered the process in which some knowledge gradually diffused among the players. He followed Hayek's view that "we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge" (Hayek 1949, p. 91). Morgenstern (1976f p. 467) suggested a thought experiment. Consider a society consisting of fully rational individuals who have limited knowledge. What will happen if *some* members of this society come to know a certain theory which predicts the outcome of their economic activities? The more widely this theory is absorbed by participants in this society, the easier it would be to predict future results of economic interaction among them. But what would happen if they come to understand that the theory's predictions would not be realized if *all* participants should behave according to the theory? Would not those who were fully rational move to behave idiosyncratically by deviating from the theory (say) in order to profit from others' behavior? The more a behavioral theory is absorbed in a society, the more difficult it would be to predict outcomes of behaviors it is supposed to explain. The logic of an economic theory is not unassailable in practice; it does not have the power to predict the results of human interaction as a logical consequence of the theory. Moreover, "the distribution of the kind of theory available, and the degree of its acceptance, will differ from one case to the other. This will in turn affect the working of the economy. There is thus a 'backcoupling' or 'feedback' between the theory and the object of the theory" (p. 467).

While Morgenstern was fascinated throughout his career with the problem of predictability, he set aside the above reservations; he accepted provisionally that a general 'solution' of a theory had to satisfy the criterion of theoretical determinateness. It is this methodological desideratum that is followed in *TGEB*; it is in accord with von Neumann's treatment of information and knowledge, and its acceptance further contributed to making the collaboration successful.

4. Conclusion

We have considered the process of collaboration towards the birth of *TGEB*, according to the development of the 'game' concept. While the general framework was already structured by the article of J. von Neumann in 1928, the ideas of O. Morgenstern were important at each stage of the development. At the first stage concerning the analysis of games of chance, where only single players were treated, and where uncertainty was considered only in connection with future events occurring with some probability, Morgenstern's analysis of expected utility with the time element was critical.

At the second stage concerning the analysis of games of strategy, it was supposed that rational players acted independently but with full knowledge of the behavior of the other

players. Here the interaction of two or more individuals was important; their interests were frequently in conflict. Here information and learning was restricted; the players had to act separately. At this stage J. von Neumann became conscious of the importance of their interaction as a game for the first time, and conceived competition as a mutual struggle. Morgenstern approached a similar problem starting with an examination of the concept of competition and then he considers the acquisition and exchange of information among the competitors. This second stage was decisive for the collaboration, but it also made clear the different ideas that the two collaborators had about a 'solution' to the game. This became more explicit at the third stage. While J. von Neumann only hinted at the importance of coalition formation, O. Morgenstern pointed out the need for an analysis different from that of the mini-max type by expressly considering the problem of information diffusion.

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Notes

1 Recently this field of research about the birth of game theory has been growing fairly rapidly. An early contributory work is a book of papers edited by E. R. Weintraub (1992). Some authors have continued research in this field (U. Rellstab (1992b), R. J. Leonard (1994), (1995), C. Schmidt (1990), (1995), A. Innocenti (1995)). Further, there is much room for additional study in connection with the history of the refinement of general equilibrium theory and with the colloquium led by K. Menger (for example, L. F. Punzo (1989), (1991)).

2 I would like to mention only one of the examples of investigation on this theme here;
"In 1657, Christiaan Huygens published ... *Calculating in Games of Chance*, in which the notions of expected value and equivalent gambles were invented... a fair price can be found for any bet by finding a fair lottery such that the bettor is indifferent between the lottery ticket for the original bet and the fair lottery" (A. E. Cudd 1993, p. 112.) .

3 Because of frequent reference to this article and the high demand to read the original by economic theorists, especially after the publication of *TGEB*, it was translated into English in 1956. But in this article it was mentioned that another mathematician, G. Cramer, had also reached a similar conclusion about the same theme a few years before D. Bernoulli did.

4 M. Suzuki (1994) writes as follows;
"The first economic theory that mentioned it (the article of D. Bernoulli) was the book by A. A. Cournot in 1838. The German translation was published in 1896. ... It may be appropriate to introduce D. Bernoulli as the precursor of the theory of marginal utility" (M. Suzuki 1994, p. 398).
In German economics, according to J. A. Schumpeter (1954, p. 303, 393), F. B. W. von Hermann (1832, Staatswissenschaftliche Untersuchungen, München) and H. von Mangoldt (1855, Die Lehre von Unternehmergewinn, Leipzig) was conspicuous in investigating the problem of risk. The former quoted D. Bernoulli which was appreciated by J. A. Schumpeter (ibid., p. 304). Streißler (1994, p. 22) pointed out that this appraisal by J. A. Schumpeter stems from secondary literature and that it is a kind of misreading of F. B. W. von Hermann. On the other hand, Fick wrote an introduction for the German translation of D. Bernoulli and mentions F. B. W. von Hermann in this connection (Fick 1967, p. 17-18).

5 As for H. von Mangoldt, he mentioned risk when considering the entrepreneur, which had not been accepted, for example, by C. Menger. Menger explained his disagreement with von Mangoldt as shown below, which proves that he did not understand von Mangoldt. H. von Mangoldt showed the 'acceptance of danger ' was the essential thing for activity of entrepreneurs (Unternehmung). But the 'danger' is, according to Menger, only accidental and the chance of gain can compensate for the loss (Menger 1933a p. 137).

6 The report had stimulated O. Morgenstern so much that he published it in the *Zeitschrift für Nationalökonomie* in 1934 after he was appointed to the editorial board (K. Menger 1979 p. 137). Further, he persuaded J. von Neumann to undertake a formal treatment of utility in *TGEB* with this article (H. W. Kuhn and A. W. Tucker 1958 p. 108). But it was only after some decades that the issue of several attitudes toward risk was much discussed in game theory.

[7] E. von Böhm-Bawerk (1881). This was his "Habilitation" (university lecturing qualification). Streißler (1969 p. 251) pointed out that Morgenstern stressed that he owed much to this work of Böhm-Bawerk.

[8] He followed the theory of C. Menger published in 1871, which also was conscious of the room for uncertainty in the classification of goods to some extent.

[9] E. Streißler (1969) argued that E. von Böhm-Bawerk was the only discoverer of the concept of certainty equivalence. Further he pointed out that the analysis of an auction process which contained some elements of a lottery had been a feature of K. H. Rau's (1841) textbook.

[10] It is said in general that the Austrian School of Economics considered the time element in the treatment of marginal utility traditionally, beginning with its introduction of C. Menger, followed by E. von Böhm-Bawerk and F. von Wieser. Further, O. Morgenstern referred to H. Mayer, who worked on planning by an individual over specified periods of time (Morgenstern 1976a).

[11] Innocenti (1995) also points this fact out and characterizes it as the principle of subjective rationality.

[12] "... in asking what a person who owns 2 splinters will give for the third splinter, he (E. von Böhm-Bawerk) neglects to consider that the behavior of the other contractor must also be investigated in this exchange" (Morgenstern 1976a, p. 155).

[13] P. A. Dimand and M. A. Dimand (1992 p. 16) explain that the a game called 'le Her' was investigated by D. Bernoulli and his friend, J. Waldegrave.

[14] E. Borel also wrote several articles on a similar subject in 1921, 1924, 1927, and proved the existence of the saddle point for some special cases of current games (such as in the game of paper, scissors, stones with two, three, and five players, which was also a popular childrens game in Japan. But he did not give any general framework for games and was rather sceptical about the general validity of the theorem. Besides, the mini-max strategy and proof of the theorem of the existence of an outcome were set forth only obscurely in his writings. So it is doubtful whether this investigation of E. Borel can count as a prior discovery. See, for example, the article of M. Fréchet (1953) and the comment by J. von Neumann (1953).

[15] J. von Neumann gave a footnote as follows: "This is the principal problem of classical economics: how is the absolutely selfish 'homo econominus' going to act under given external circumstances?" (J. von Neumann 1928; 1959 p. 13).

[16] The first type of equilibrium theory without the assumption of perfect foresight was treated by L. Walras, V. Pareto and H. von Stackelberg, while J. R. Hicks, G. Cassel, F. A. Hayek treated the second type. Morgenstern wrote another article criticizing J. R. Hicks on this point (Morgenstern [1976d]), and he also offered some criticisms of Cassel which were not published, "Bemerkungen zu Cassel's Preistheorie (Remarks on Cassel's price theory)" (U. Rellstab 1992b, pp. 156-159). In a similar manner, Leonard (1995, p. 755) argued that *TGEB* had also intended to challenge radically the Hicks-Samuelson variant of neoclassical economics. Another type of equilibrium theory, based on Nash equilibria, also contains specific assumptions about foresight and rationality. These deserve investigation by historians of economic thought.

[17] Morgenstern recalled a report which led him to recognize the similarity with the work of von Neumann. "... after the meeting broke up, a mathematician named Eduard Cech came up to me and said that the questions I had raised were identical with those dealt with by John von Neumann in a paper on the Theory of Games published in 1928... Cech ... outlined to me its principal ideas and results and was very eager that I should study this particular work" (Morgenstern 1976c, p. 806). But it was only several years later that they met, namely in Princeton in America, not in Vienna. Only then did their collaboration really begin.

[18] As Rellstab (1992b, p.53) points out, there is no conclusive direct textual evidence for the thesis that Morgenstern was dissatisfied with the static nature of game theory in particular. However, there is extensive indirect evidence, especially in Morgenstern's analysis of the role of time in economic theory and in his sceptical attitude toward the notion of perfect foresight.

[19] The term 'bounded rationality' was coined by H. A. Simon who also studied early work on game theory closely. He claimed that there was some similarity between bounded rationality and the viewpoint of the Austrian School of Economics on the subject, especially Hayek's. "I realized that ... one could think of the Austrian viewpoint, particularly von Hayek's version of it, as a form of bounded rationality. Among other things, the Austrians put a tremendous emphasis on tacit and personal knowledge... " (H. A. Simon et al. 1992, p. 26).

[20] See also Innocenti (1995, p. 217) on this point.

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